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UNIVERSITY OF OXFORD

Fermion Hierarchies from String Theory

Lucas Leung

based on arXiV:2410.17704 and arXiV:2507.XXXXX
In collaboration with: Andrei Constantin, Kit Fraser-Taliente, Thomas Harvey, Andre Lukas and Luca Nutricati

International School of Subnuclear Physics 2025 Erice - 20th June 2025

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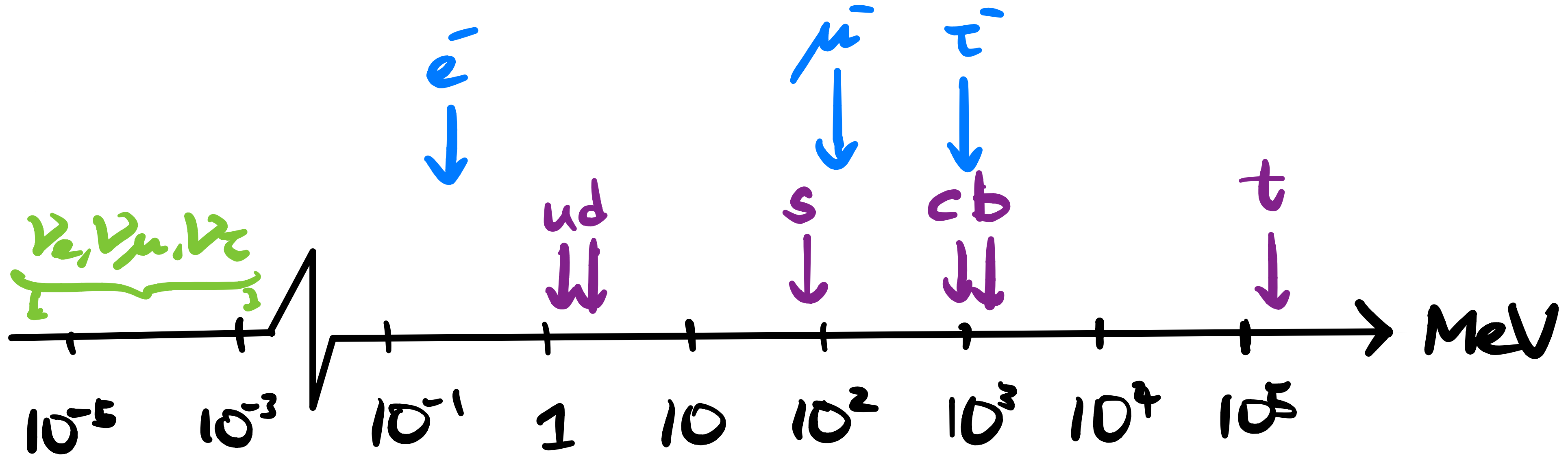
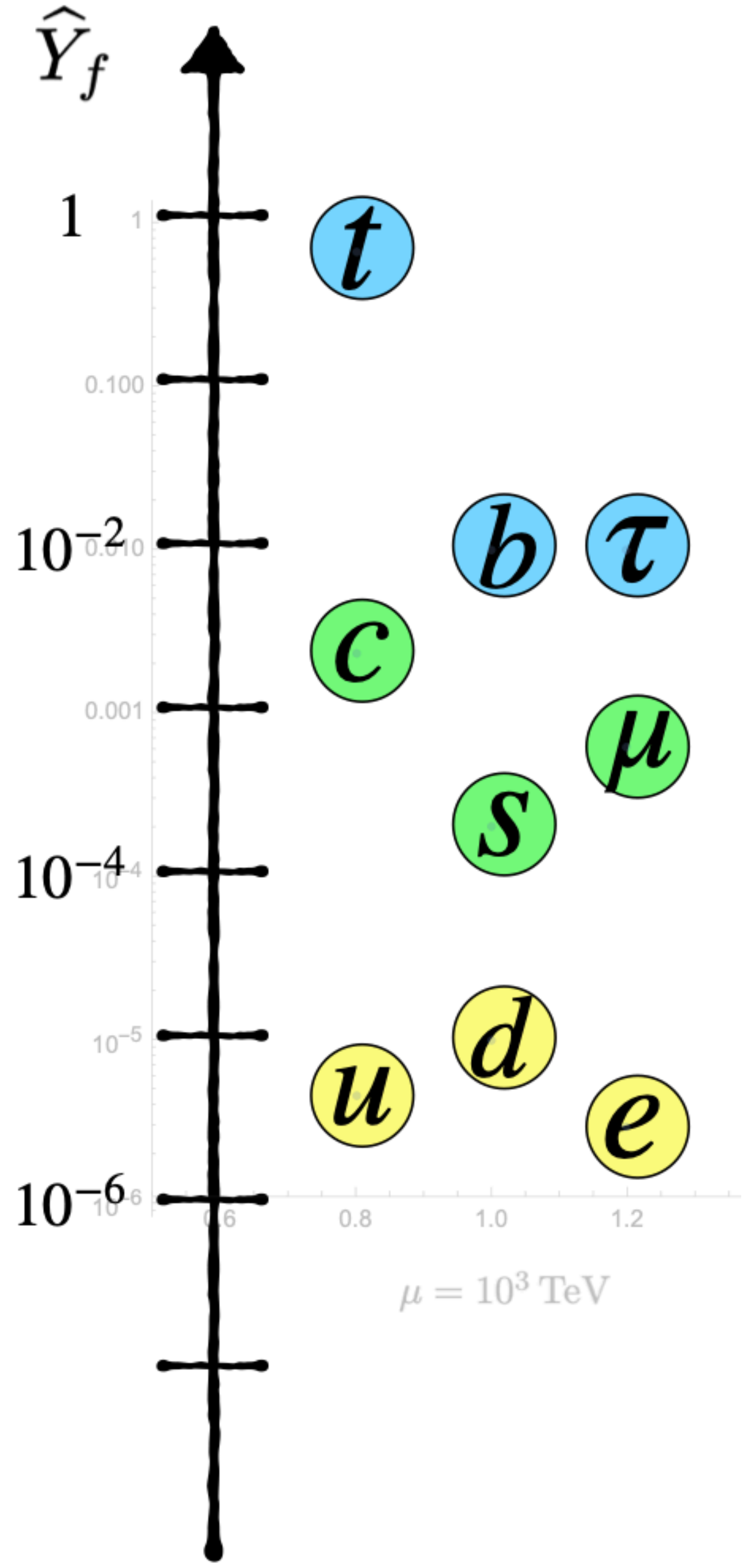
Fermion Masses and Mixings

Why do we have fermion mass hierarchies?





$$V_{\text{CKM}} \sim \begin{pmatrix} 1 & 0.2 & 0.2^3 \\ 0.2 & 1 & 0.2^2 \\ 0.2^3 & 0.2^2 & 1 \end{pmatrix}$$

stolen from Admir Greljo's La Thuile and Matthew McCullough's Erice talk

Froggatt-Nielsen Mechanism

- **Froggatt and Nielsen** [1979] proposed using horizontal symmetries $U(1)_H$ to explain flavour structures

O(1) coefficient

$$y_{ij} = a_{ij} \langle \phi \rangle^{n_{ij}}$$

$U(1)_H$ **neutral**

Q_1	Q_2	Q_3	$\bar{d}_1$	$\bar{d}_2$	$\bar{d}_3$	$\bar{u}_1$	$\bar{u}_2$	$\bar{u}_3$
(3)	(2)	(0)	(3)	(2)	(2)	(3)	(1)	(0)

$+$

ϕ
(-1)

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Set $\langle \phi \rangle \sim \lambda$

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$$M^d \sim \langle H_d\rangle \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix}, \quad M^u \sim \langle H_u\rangle \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^3 \\ \lambda^5 & \lambda^3 & \lambda^2 \\ \lambda^3 & \lambda & 1 \end{pmatrix}.$$

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$\phi^6 Q_1 H_d \bar{d}_1$

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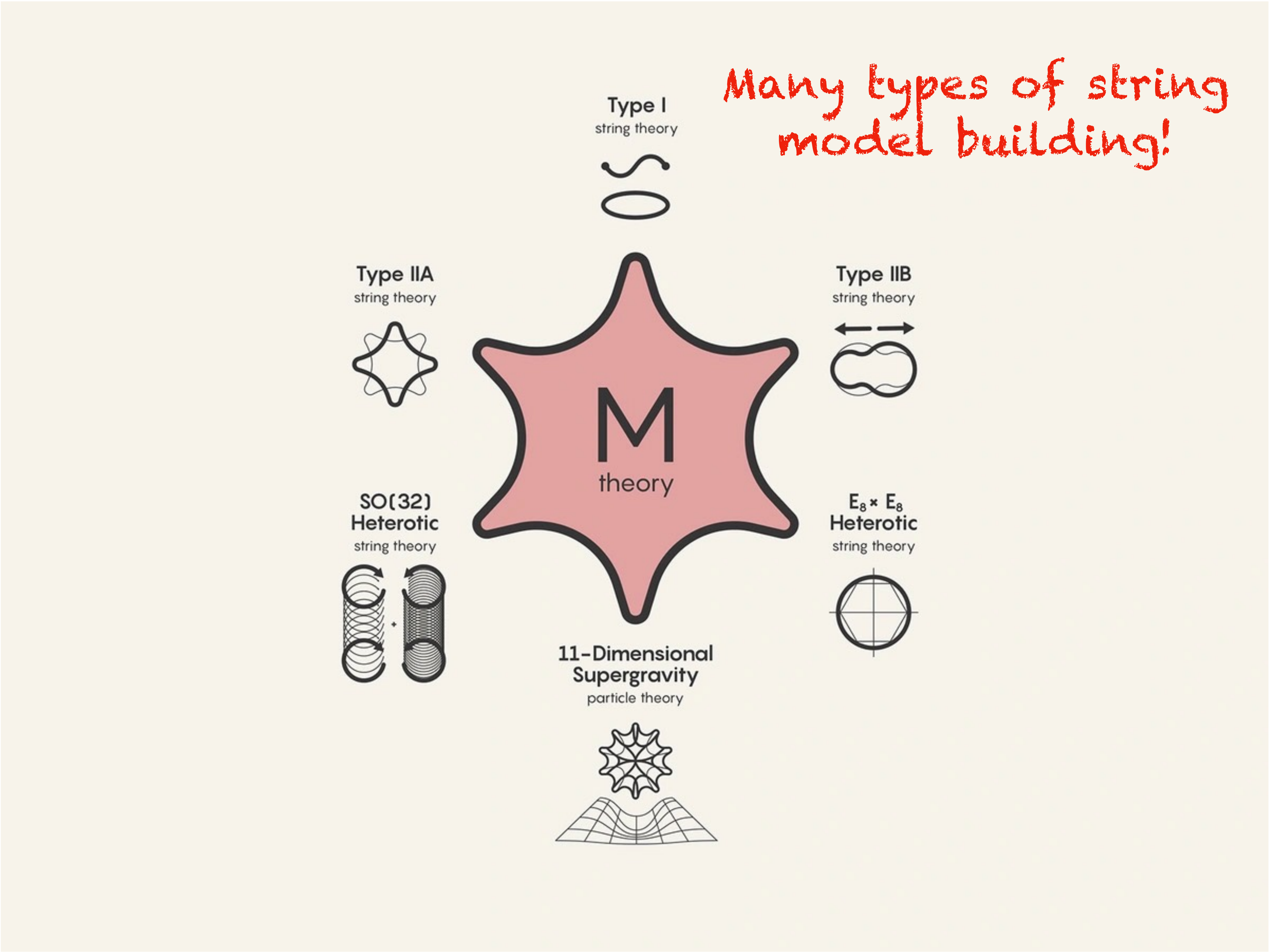
String Model Building

- most well-studied QG theory
- maybe best theory to understand HEP



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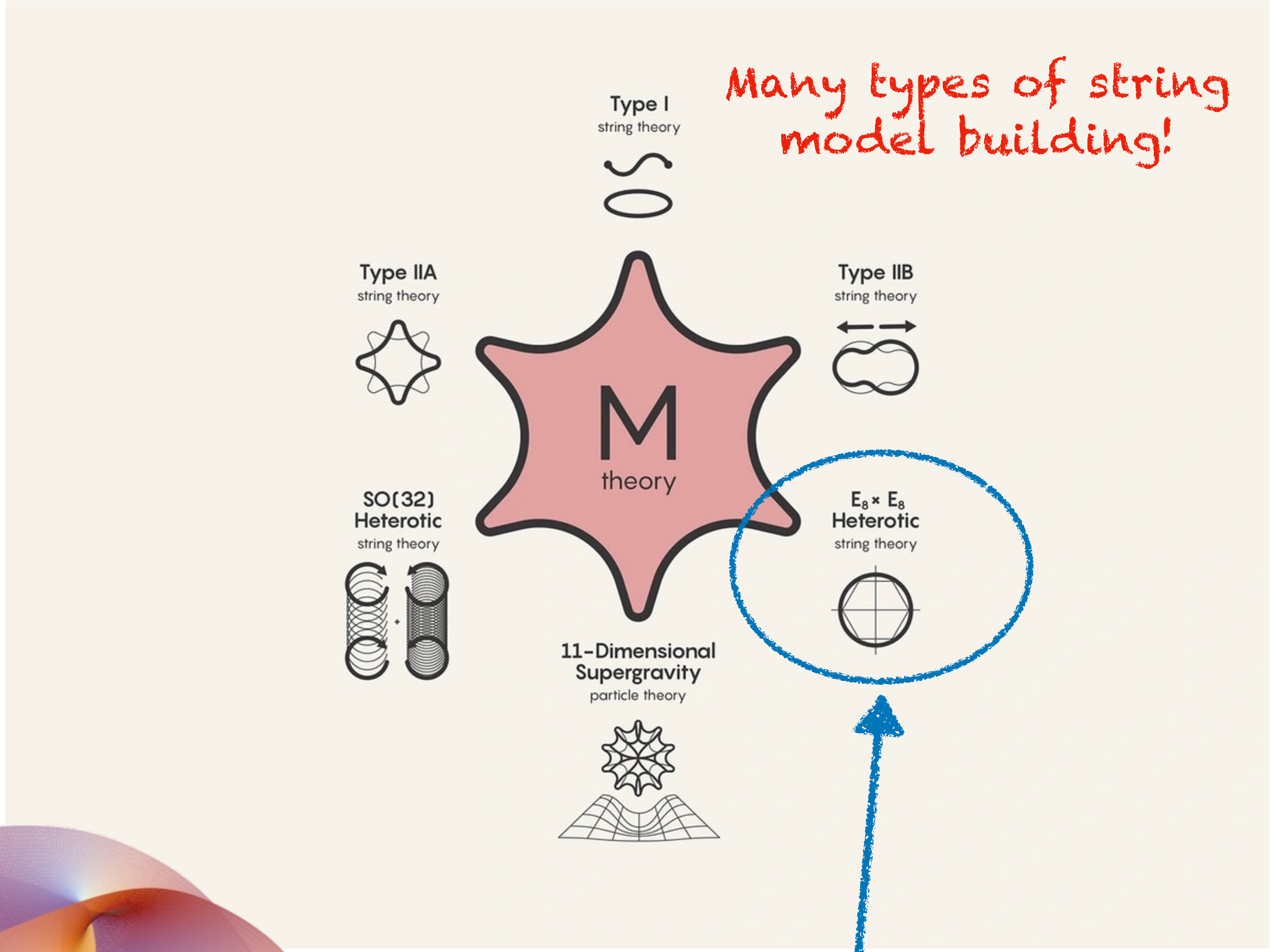
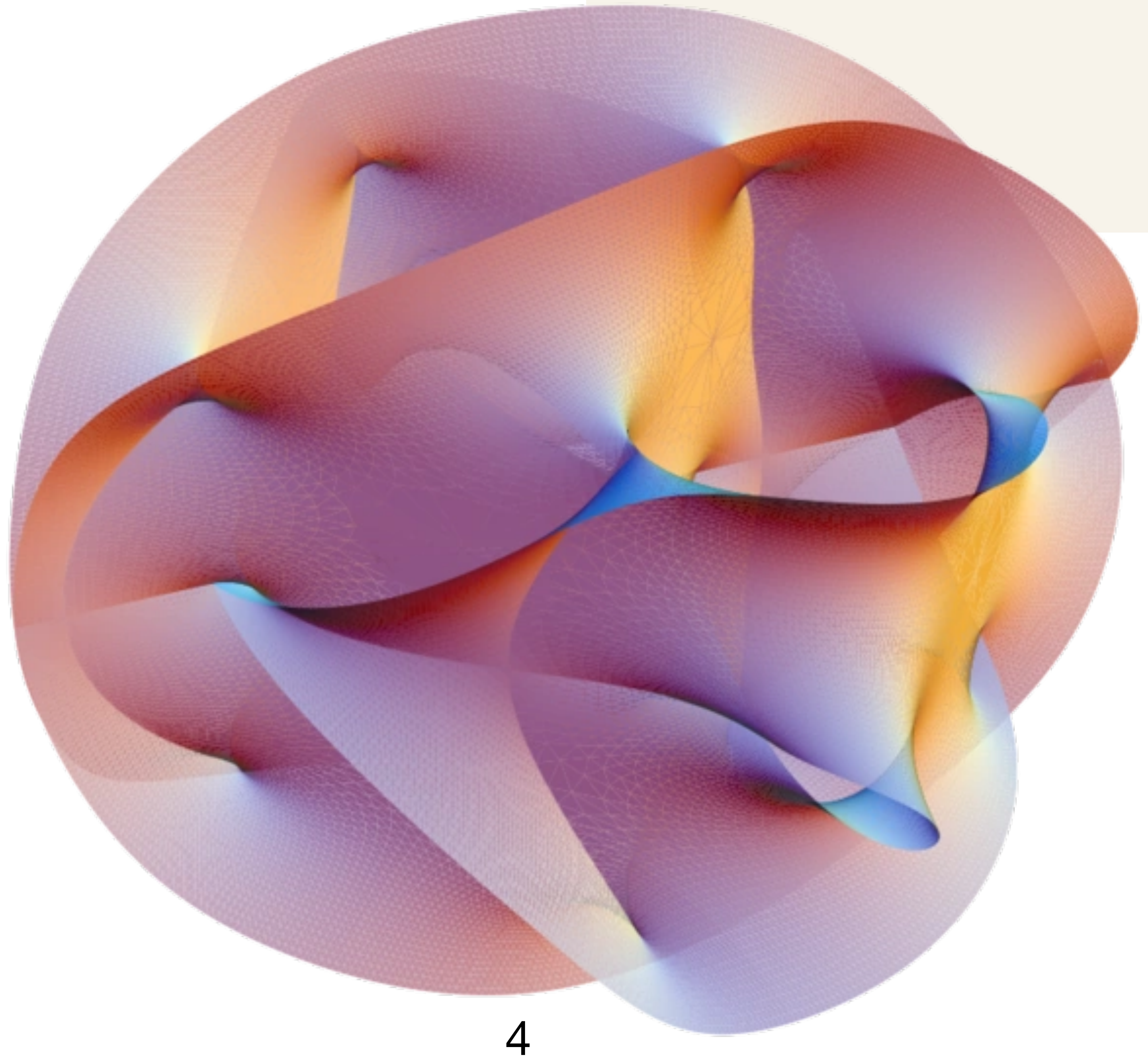


Many types of string model building!



Want to focus on heterotic $E_8 \times E_8$ on smooth CYs

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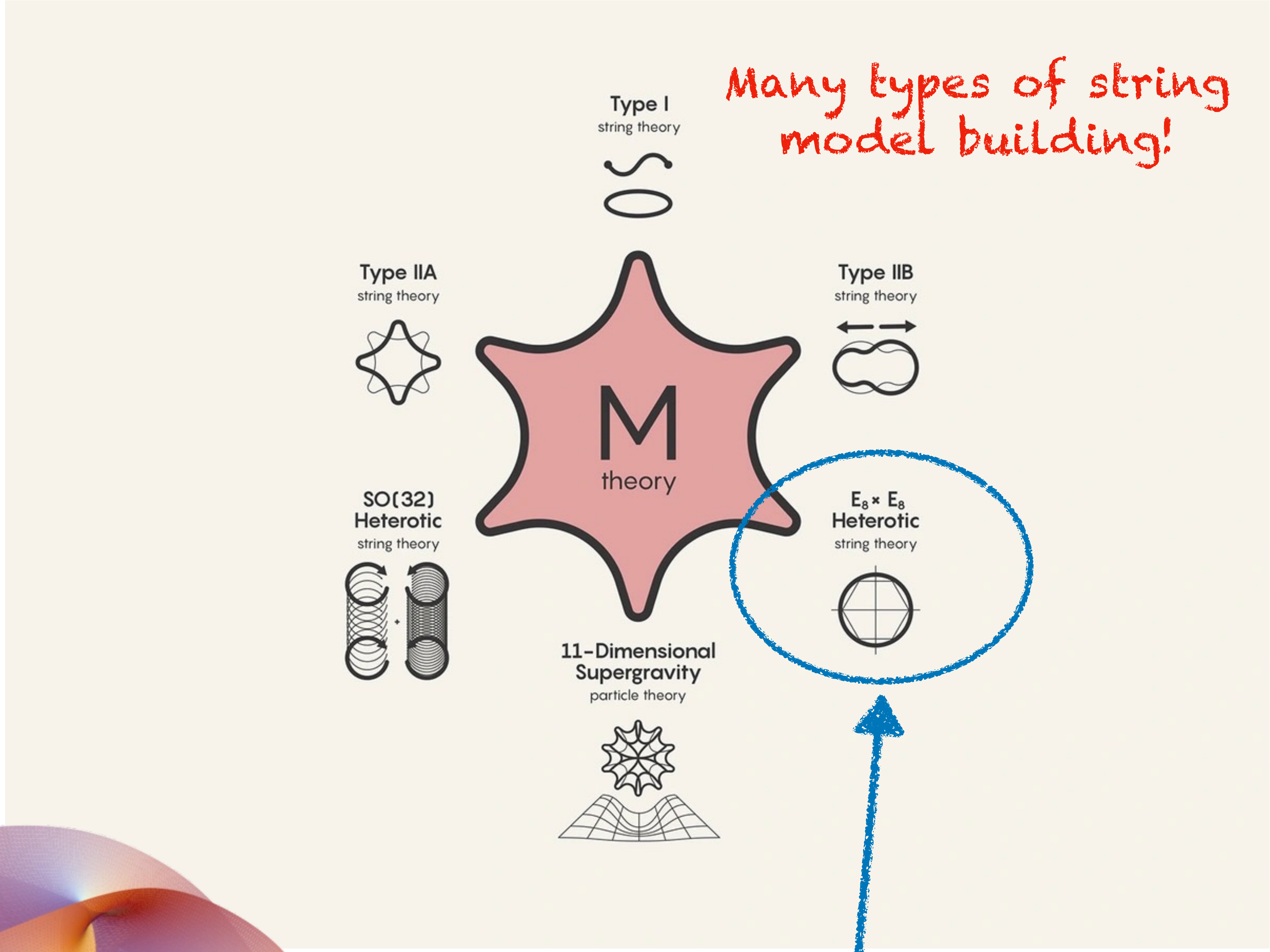

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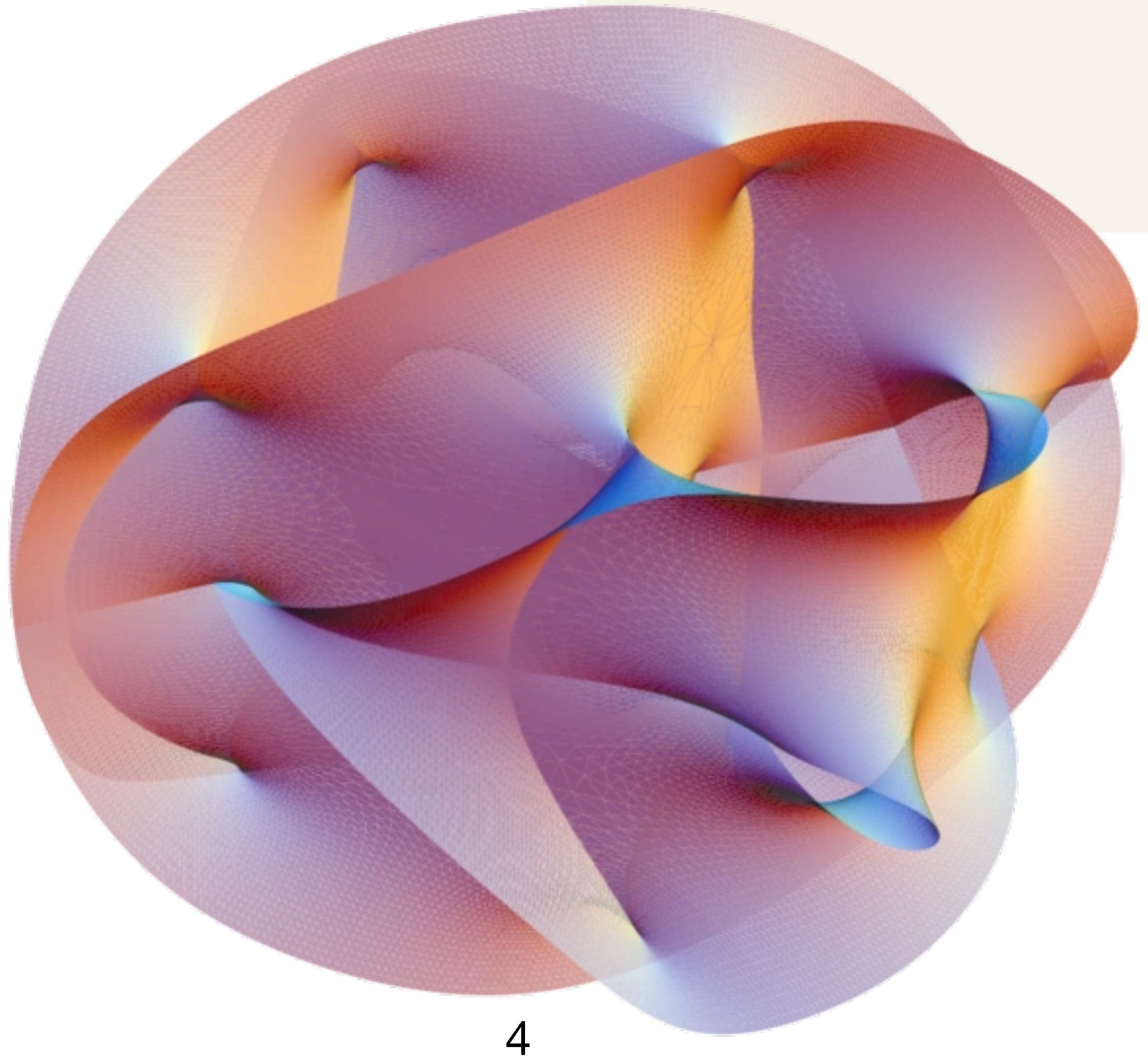
Why Strings?

Intuition to how different 'nuts and bolts' in a HE theory 'interact' to give Standard Model + answer other pheno questions.



U(1) symmetries from string theory

- Low-energy effective theories have gauge symmetry

$$SU(3)_c \times SU(2)_L \times U(1)_Y \times \mathcal{G}$$

where $\mathcal{G} \cong U(1)^n/\mathbb{Z}n$

field	SM rep	name	SU(5)	$\mathcal{G}$ charge pattern	SU(5)$\times\mathcal{G}$
Q	$(\mathbf{3},\mathbf{2})_1$	LH quark	$\mathbf{10}$	$\mathbf{e}_a$	$\mathbf{10}_a$
u	$(\bar{\mathbf{3}},\mathbf{1})_{-4}$	RH u-quark			
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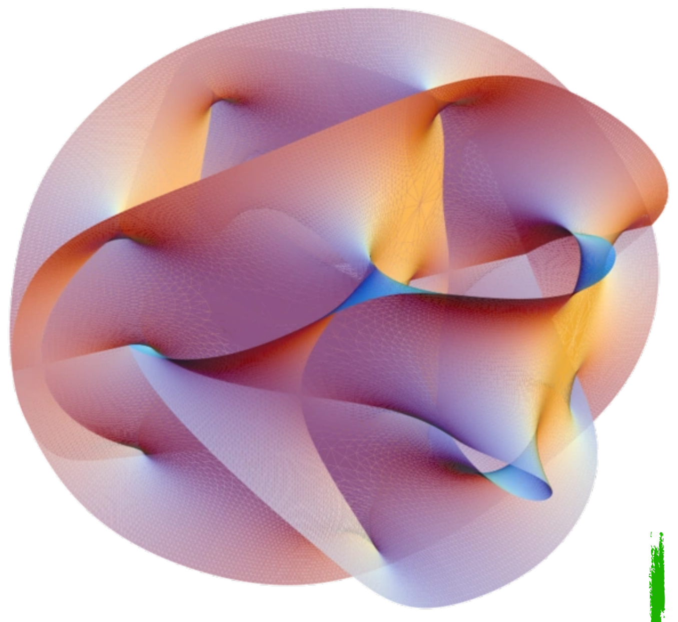
2 - Specific Charges for scalar + SM fields

Goal

- Want to understand **fermion hierarchies** using **U(1)s from string theory**



String Theory

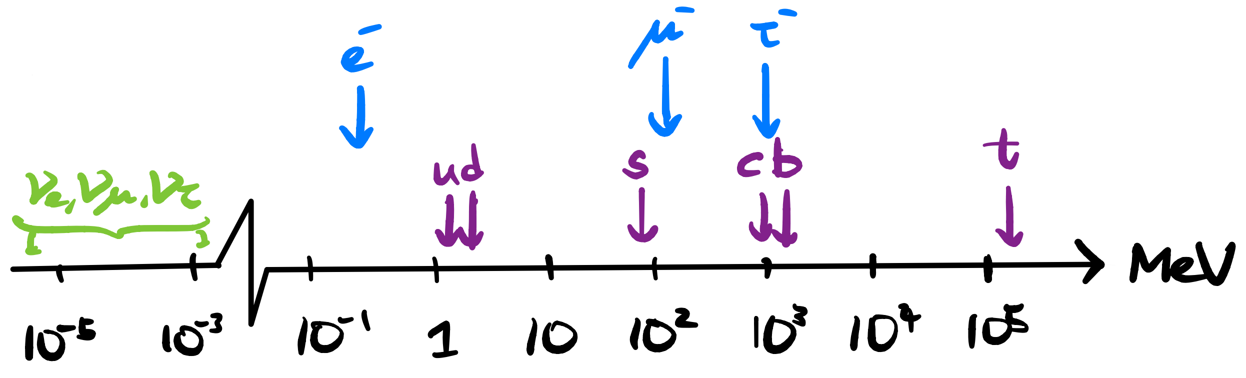
Top-down Approach

- Use symmetry patterns of known string models - compute mass hierarchies
- **Difficult - satisfy other phenomenological properties (Higgs? Neutrinos?)!**

Bottom-up Approach

- Use charge patterns from string theory as constraints of effective theory
- **Large number of possible patterns - analyse using Machine Learning & Genetic Algorithms**

Fermion Hierarchies



Examples

Top-down Approach

Downstairs Spectrum

Matter fields	$\mathbf{10}_1,\ \mathbf{10}_2,\ \mathbf{10}_5,\ \overline{\mathbf{5}}_{1,3},\ \overline{\mathbf{5}}_{2,3},\ \overline{\mathbf{5}}_{4,5}$
Higgs fields	$H_{3,5},\ \overline{H}_{3,5}$
Moduli	$\Phi_1,\ \ldots,\ \Phi_5$
	$\phi_{1,2},\ \phi_{2,1},\ \phi_{1,4},\ \phi_{5,1},\ \phi_{2,3},$
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Yukawa Insertions

$$\Lambda^u \sim \begin{pmatrix} \phi_{3,5}\,\phi_{5,1}^2 & \phi_{2,1}\,\Phi_2 & \phi_{3,5}\,\phi_{5,1} \\ \phi_{2,1}\,\Phi_2 & \Phi_2 & \phi_{2,5}\,\Phi_2 \\ \phi_{3,5}\,\phi_{5,1} & \phi_{2,5}\,\Phi_2 & \phi_{3,5} \end{pmatrix}$$

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Examples

1 - Models exist!

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Examples

1 - Models exist!

3 - charge constraints!

2 - a lot of extra fields!

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Link?

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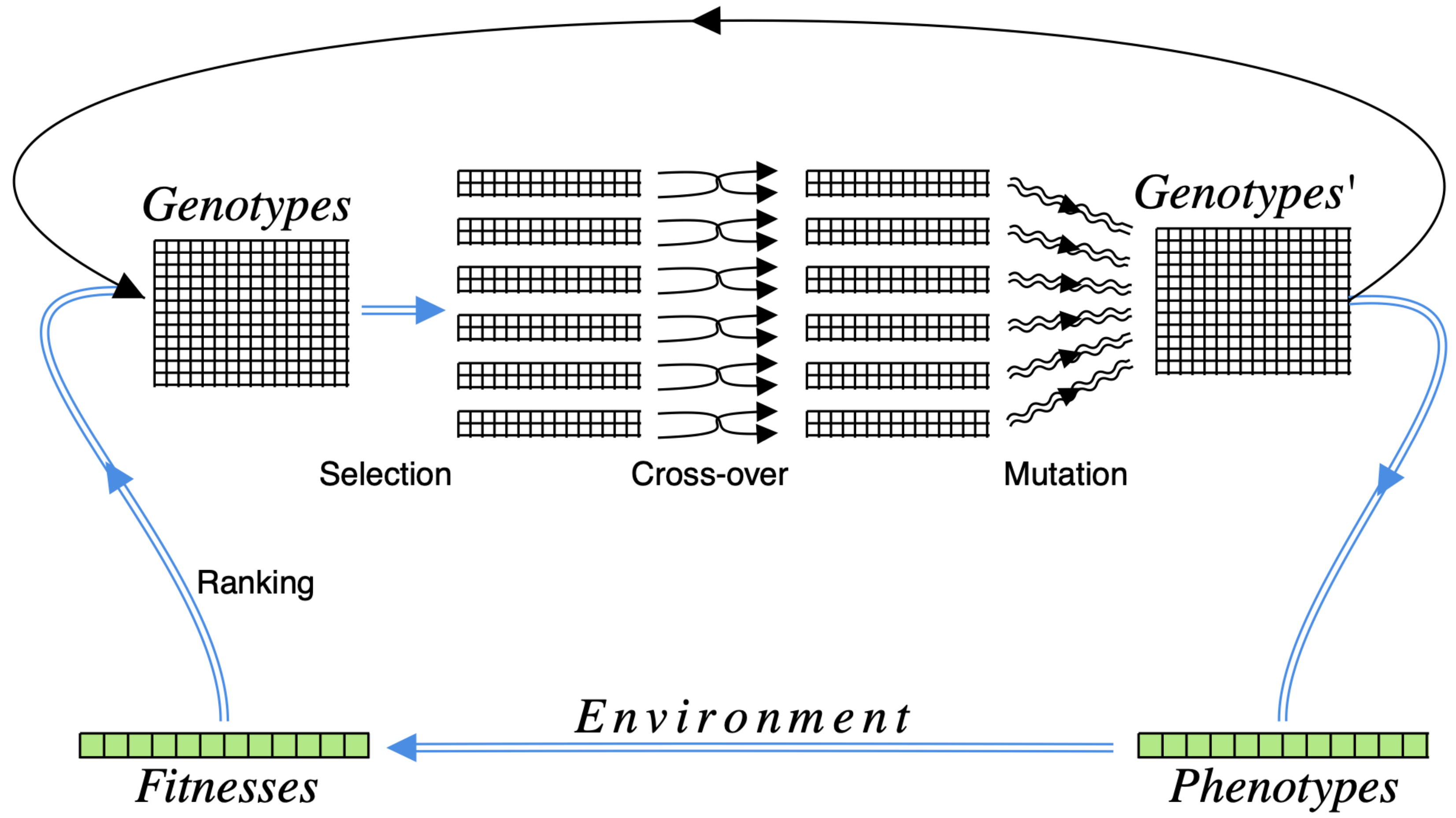
Backup Slides

Genetic Algorithms

- A family of optimisation-search algorithms.
- Two parts: **Environment** + **Evolution**

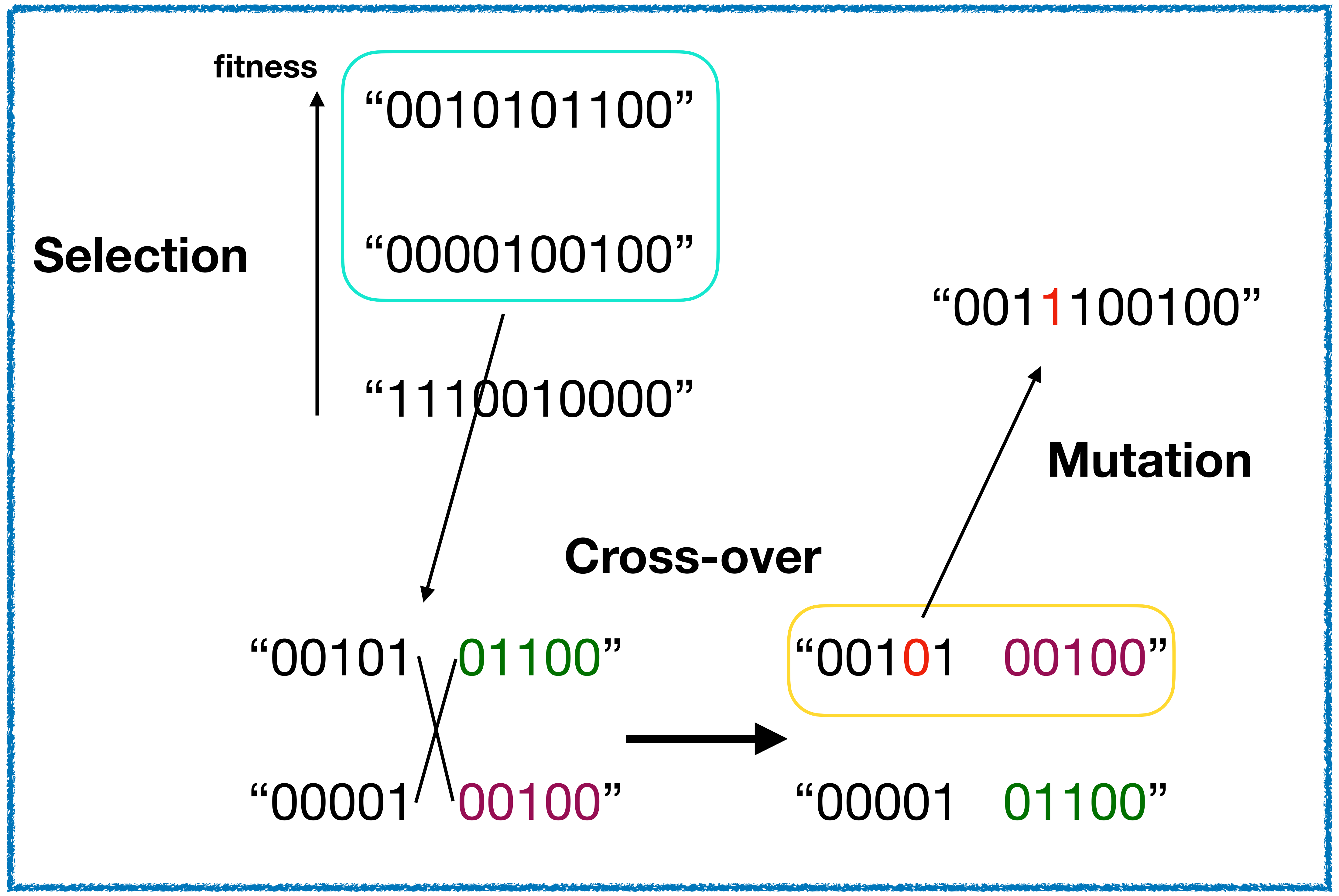


from Abel et al. ('23)

Environment

"0010101100" →	$m_u = 0.002$ →	0
"0000100100" →	$m_u = 0.03$ →	-1.18
"1110010000" →	$m_u = 150$ →	-4.88
Genotype	**Phenotype**	**Fitness**

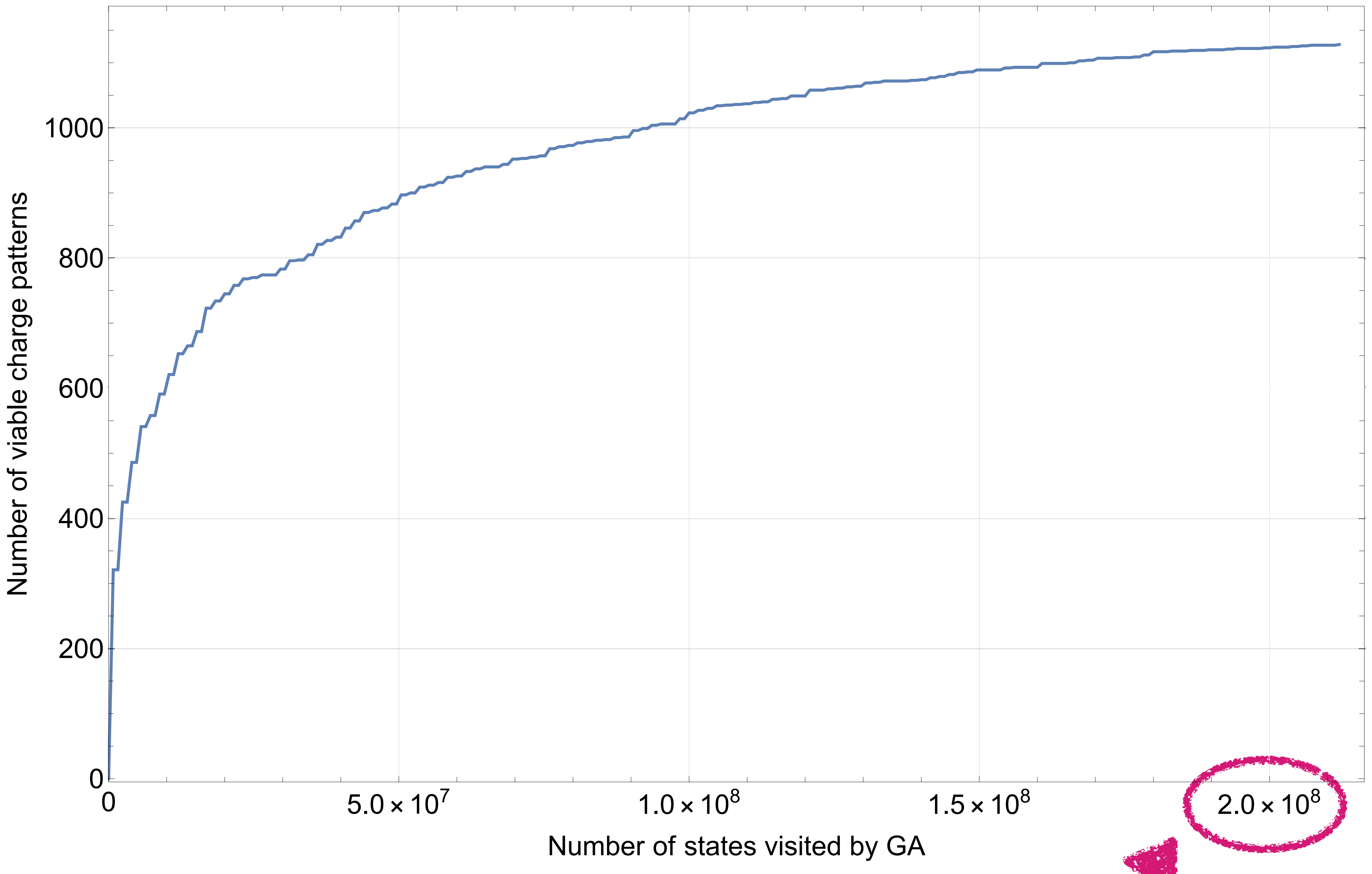
Evolution



Bottom-up Results - Scans (Perturbative Only)

n	Fixed charges	N_ϕ	Env Size	States visited	Full scan	Models	Inequiv. spectra
$(1,1,1,1,1)$	$\mathbf{10}_1, \mathbf{10}_2, \mathbf{10}_5, \bar{\mathbf{5}}^H_{4,5}$	2	10^9		Yes	0	0
		3	10^{11}	10^8		0	0
		4	10^{13}	10^8		301	4
		5	10^{16}	10^9		29213	289
$(1,1,1,2)$	$\mathbf{10}_1, \mathbf{10}_2, \mathbf{10}_4, \bar{\mathbf{5}}^H_{4,4}$	2	10^8		Yes	0	0
		3	10^{10}		Yes	98	1
		4	10^{12}	10^8		18825	55
		5	10^{14}	10^8		320557	449
$(1,1,1,2)$	$\mathbf{10}_2, \mathbf{10}_3, \mathbf{10}_4, \bar{\mathbf{5}}^H_{1,4}$	2	10^8		Yes	0	0
		3	10^{10}		Yes	56	1
		4	10^{12}	10^8		11538	128
		5	10^{14}	10^8		259175	1128
$(1,1,1,2)$	$\mathbf{10}_1, \mathbf{10}_2, \mathbf{10}_3, \bar{\mathbf{5}}^H_{1,4}$	2	10^8		Yes	0	0
		3	10^{10}		Yes	70	3
		4	10^{12}	10^8		8110	63
		5	10^{14}	10^8		204148	500
$(1,1,1,2)$	$\mathbf{10}_1, \mathbf{10}_3, \mathbf{10}_4, \bar{\mathbf{5}}^H_{1,2}$	2	10^8		Yes	0	0
		3	10^{10}		Yes	0	0
		4	10^{12}	10^8		0	0
		5	10^{14}	10^8		0	0
$(1,1,3)$	$\mathbf{10}_1, \mathbf{10}_2, \mathbf{10}_3, \bar{\mathbf{5}}^H_{3,3}$	2	10^6		Yes	8	1
		3	10^8		Yes	1218	18
		4	10^{10}		Yes	22734	81
		5	10^{12}	10^8		154532	234
$(1,2,2)$	$\mathbf{10}_1, \mathbf{10}_2, \mathbf{10}_3, \bar{\mathbf{5}}^H_{3,3}$	2	10^6		Yes	0	0
		3	10^8		Yes	0	0
		4	10^{10}		Yes	0	0
		5	10^{12}	10^8		0	0

Total number of inequiv. spectra obtained against states visited



Environment size
$\sim \mathcal{O}(10^{14})$

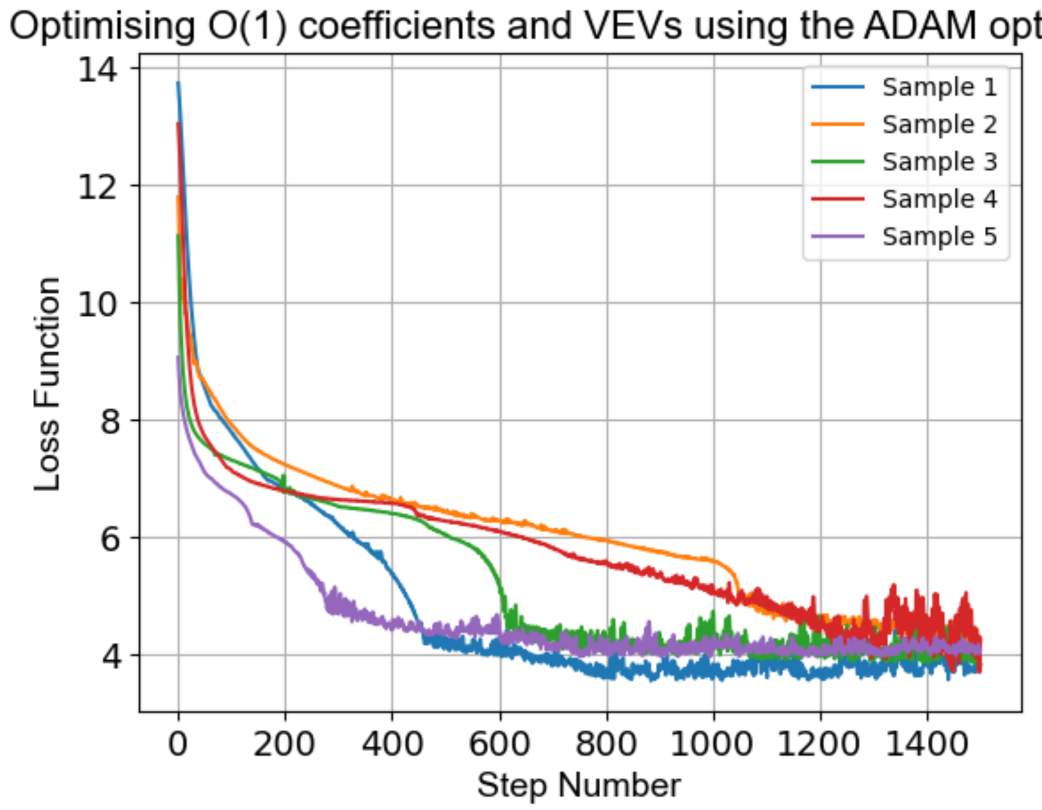
Bottom-up Example

Spectrum

$$\mathbf{10}_1,\ \mathbf{10}_2,\ \mathbf{10}_5;\ \ 3\bar{\mathbf{5}}_{1,2};\ \ H^u_{4,5},\ H^d_{4,5}$$
$$\phi_{5,1},\ \phi_{3,5},\ \phi_{1,2},\ \phi_{4,1},\ \phi_{4,5}$$

Optimise coefficients - $\epsilon = 0.554$ and

$$\underset{\text{up-quark sector}}{\begin{pmatrix} 1.090 & 2.282 & 1.896 \\ 0.961 & 2.027 & 1.979 \\ 1.966 & 2.978 & 2.648 \end{pmatrix}} \quad \underset{\text{down-quark sector}}{\begin{pmatrix} 1.379 & 1.843 & 0.947 \\ 2.708 & 1.726 & 2.063 \\ 1.530 & 2.526 & 0.680 \end{pmatrix}} \quad \underset{\text{lepton sector}}{\begin{pmatrix} 1.064 & 2.051 & 1.707 \\ 1.183 & 2.628 & 2.262 \\ 0.514 & 1.623 & 0.706 \end{pmatrix}}$$



Yukawa Textures

$$\text{up sector:} \quad \begin{pmatrix} \phi_{5,1}\phi_{4,1} & \phi_{5,1}\phi_{1,2}\phi_{4,1} & \phi_{4,1} \\ \phi_{5,1}\phi_{1,2}\phi_{4,1} & \phi_{5,1}\phi_{1,2}^2\phi_{4,1} & \phi_{1,2}\phi_{4,1} \\ \phi_{4,1} & \phi_{1,2}\phi_{4,1} & \phi_{4,5} \end{pmatrix}$$

$$\text{down sector:} \quad \begin{pmatrix} \phi_{5,1}\phi_{3,5} & \phi_{5,1}\phi_{3,5} & \phi_{5,1}\phi_{3,5} \\ \phi_{5,1}\phi_{3,5}\phi_{1,2} & \phi_{5,1}\phi_{3,5}\phi_{1,2} & \phi_{5,1}\phi_{3,5}\phi_{1,2} \\ \phi_{3,5} & \phi_{3,5} & \phi_{3,5} \end{pmatrix}$$

Compute Quantities

Higgs VEV	$\langle H \rangle = 174$ GeV		
Quark	m_u (MeV)	m_c (GeV)	m_t (GeV)
Mass	2.16	1.27	173
Quark	m_d (MeV)	m_s (MeV)	m_b (GeV)
Mass	4.70	93.9	4.18
Lepton	m_e (MeV)	m_μ (MeV)	m_τ (GeV)
Mass	0.511	106	1.78

$$|V_{\text{CKM}}| \simeq \begin{pmatrix} 0.970 & 0.242 & 0.00359 \\ 0.242 & 0.969 & 0.0447 \\ 0.00733 & 0.0443 & 0.999 \end{pmatrix}$$