Croucher Foundation
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Heterotic Line Bundle Standard Models and Heterotic Flux Moduli Stabilisation

The gift, the curse, the dream

Lucas Leung

based on arXiV:2505.XXXXX with Andrei Constantin, Andre Lukas and Luca A. Nutricati
and arXiV:2507.XXXXX with Andrei Constantin and Andre Lukas

Oxford Dalitz Seminar - 15th May 2025

UNIVERSITY OF
OXFORD

Phenomenological Questions

Fermion Masses and Mixings

Why do Yukawa matrices take these values?

Why do we have mass hierarchies?

Dark matter

What is dark matter comprised of?

Hierarchy Problem

Why is the Higgs mass/ electroweak-scale suppressed?

Strong CP Problem

Why do we have a θ-vacua?

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Fermion Masses and Mixings

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Strong CP Problem

Why do we have a θ-vacua?

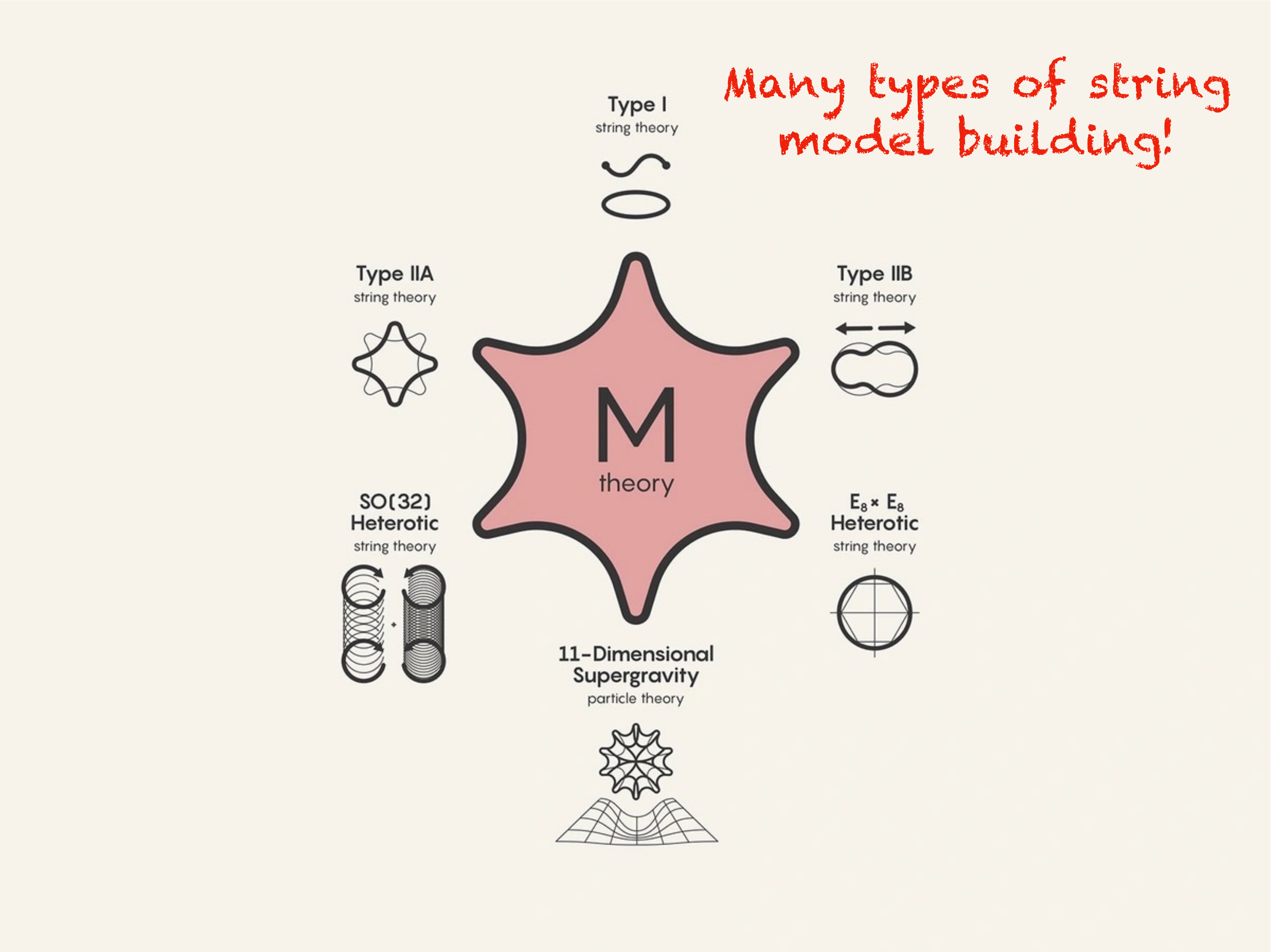
The optimistic HEP's view:

How do I unify gravity with SM?

QUANTUM GRAVITY?

String Model Building

- most well-studied QG theory
- lots of dualities
- maybe best theory to understand HEP
- can get pheno properties!



Many types of string model building!

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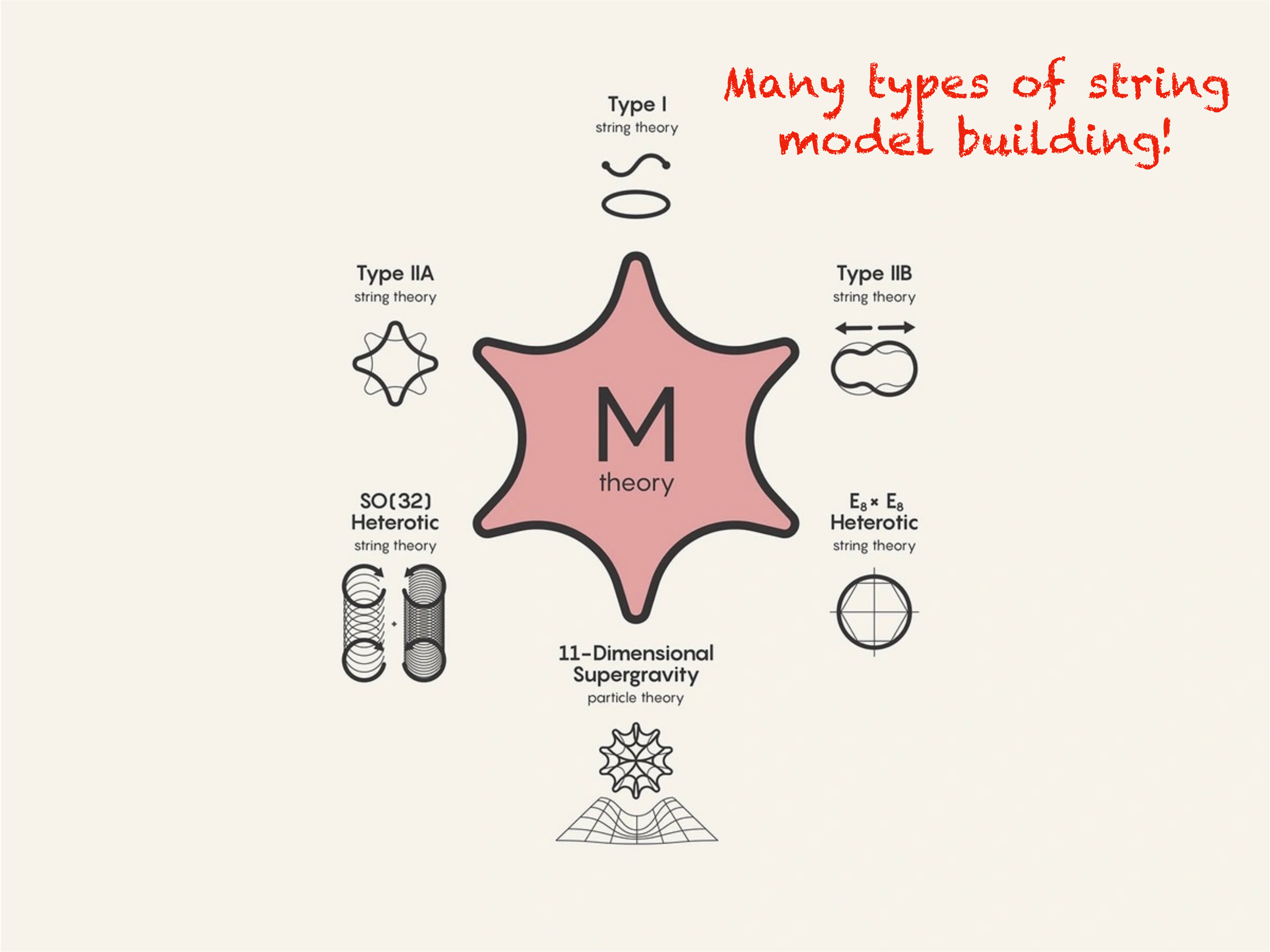

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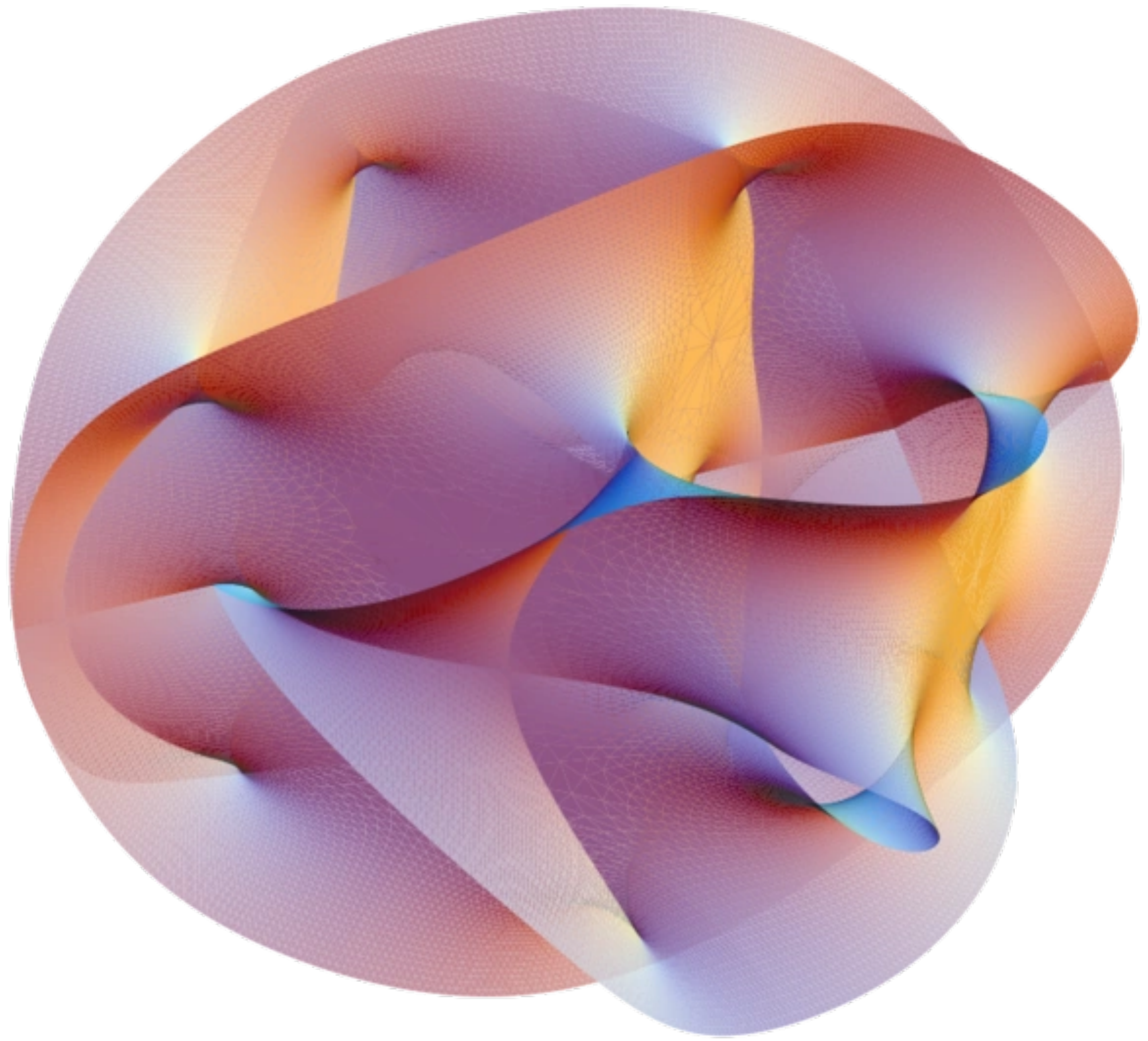
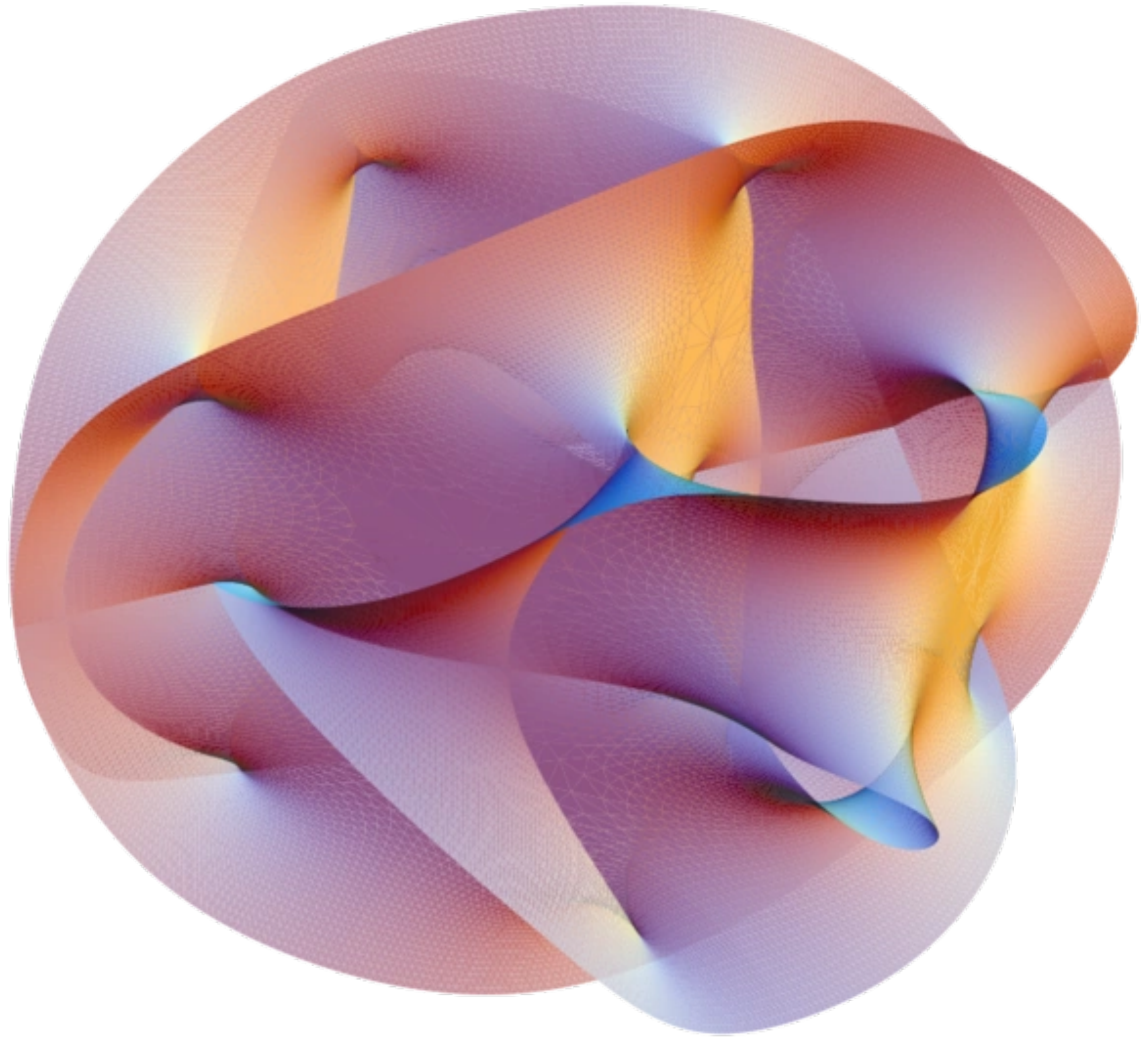


Want to focus on heterotic $E_8 \times E_8$ on smooth CYs

String Model Building



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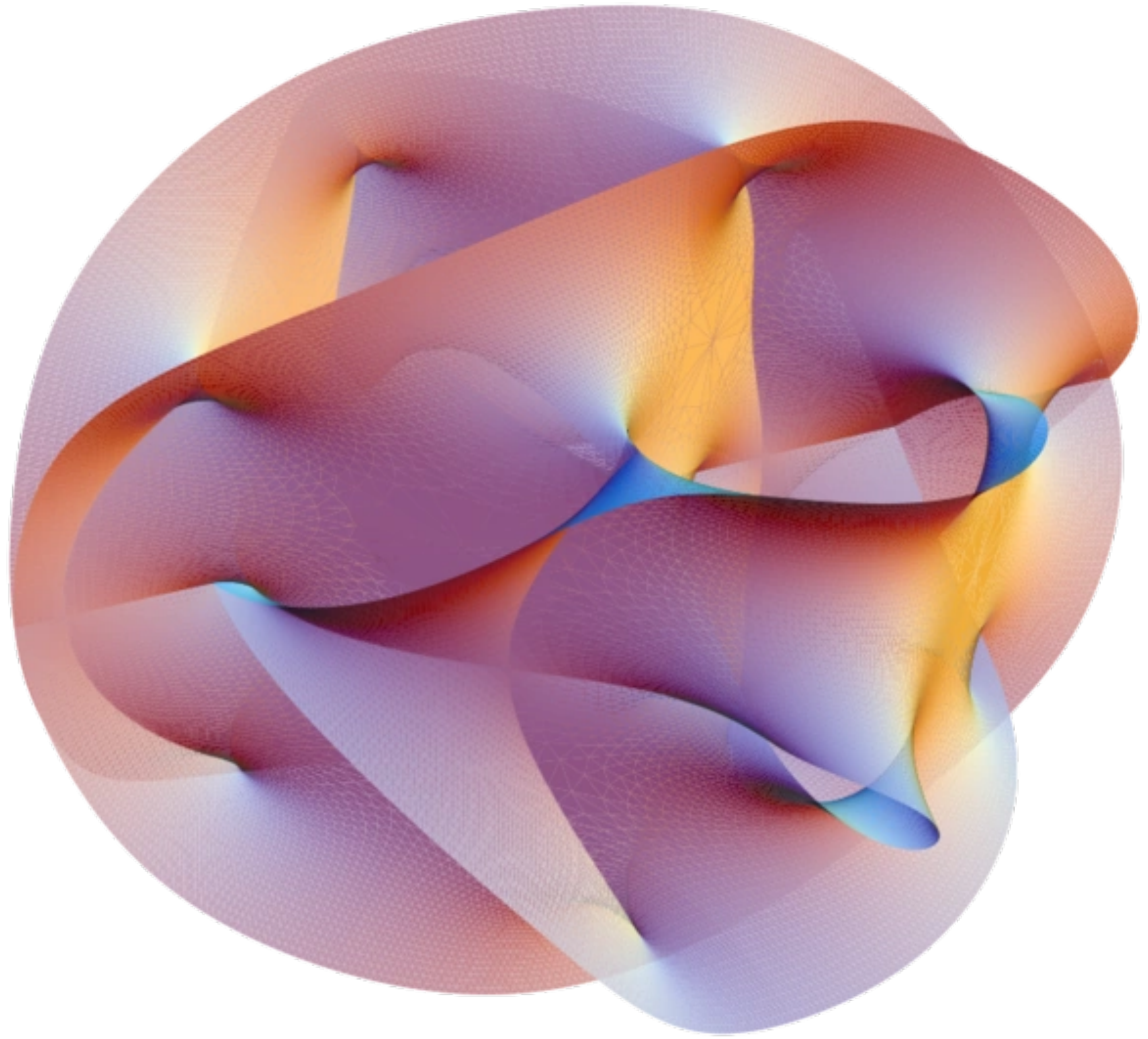

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Advantages

- Natural embedding of Standard Model gauge group into E_8
- Have lists of smooth CYs - good understanding
- Models with no exotic matter can be found

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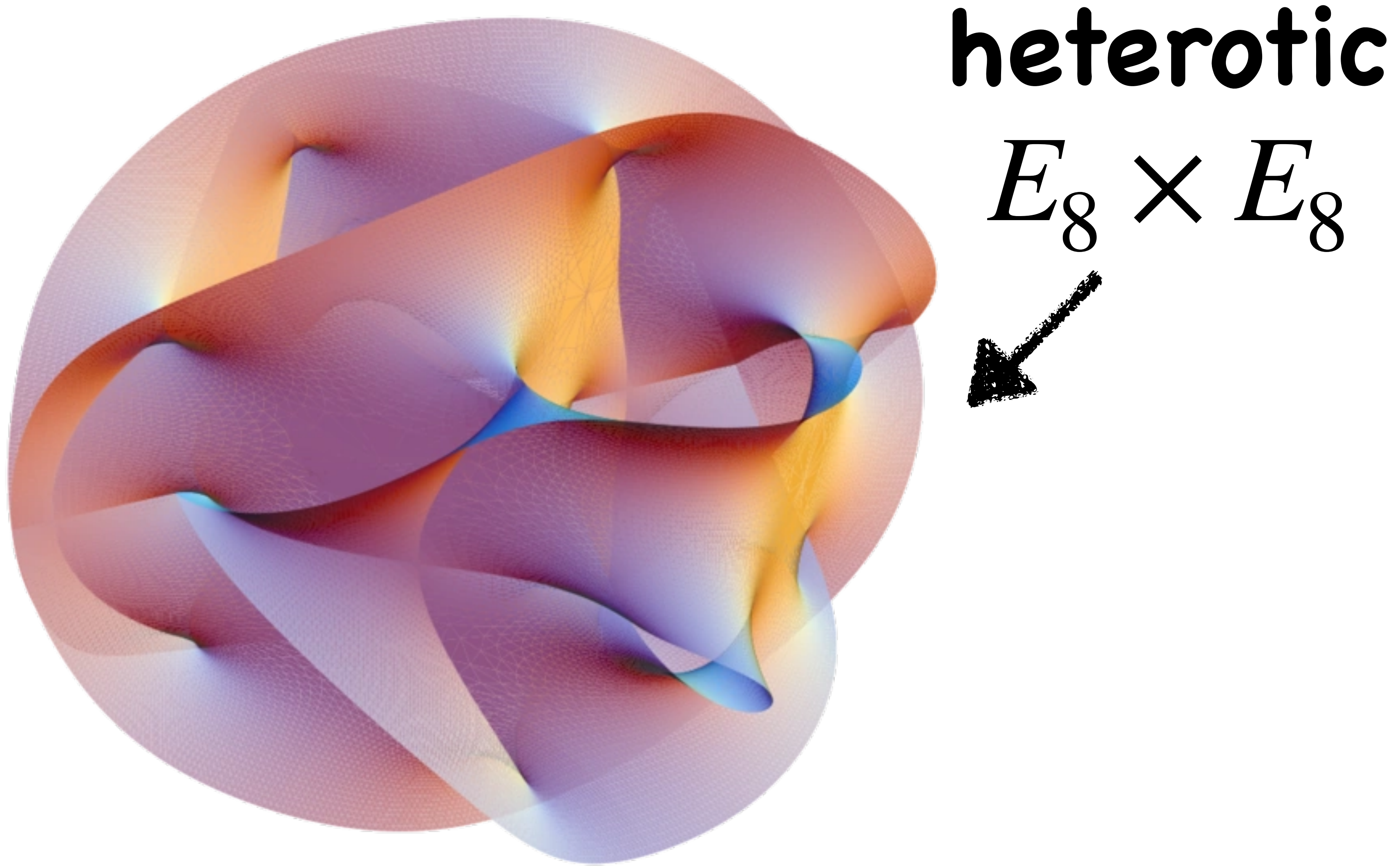

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Difficulties

- Need vector bundle - mathematically difficult
- CY metric, HYM connection not known - analytical calculations impossible
- no natural hierarchy of localisation of gauge degrees of freedom along the gravitational branes

Content

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- Heterotic Model Building and Line Bundle Standard Models
- Phenomenology of Heterotic Line Bundle Standard Models gift
- An Example Model
- Moduli Stabilisation - Strings and Heterotic
- Heterotic Flux Stabilisation - Generalities
- Example Cases and General Arguments
- Conclusions

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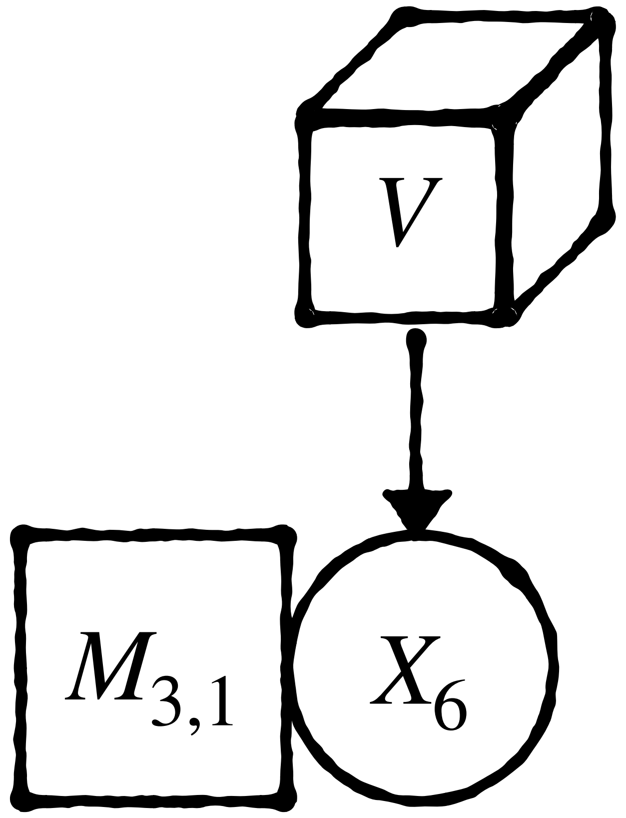
Ingredients:

- heterotic $E_8 \times E_8$ superstring theory
- Calabi-Yau threefold X
- vector bundle $V \to X$ for vector multiplets



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Topology	Spectrum
Geometry	Couplings

Step 1 : GUT Gauge Group

Step 2 : Wilson-line breaking

Step 3 : $U(1)$ symmetries

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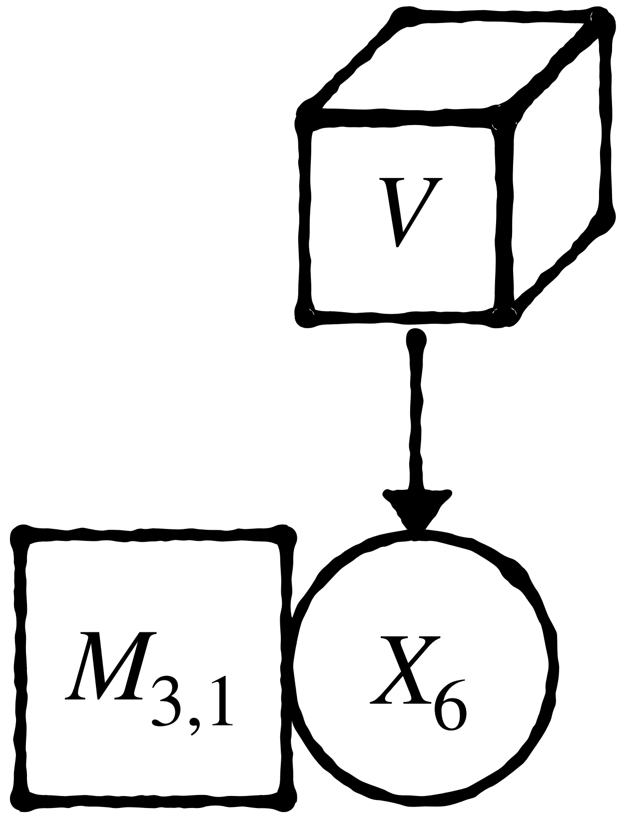
We work with **Complete Intersection Calabi-Yau manifolds (CICYs)**:

These are manifolds that are hypersurfaces in projective spaces.



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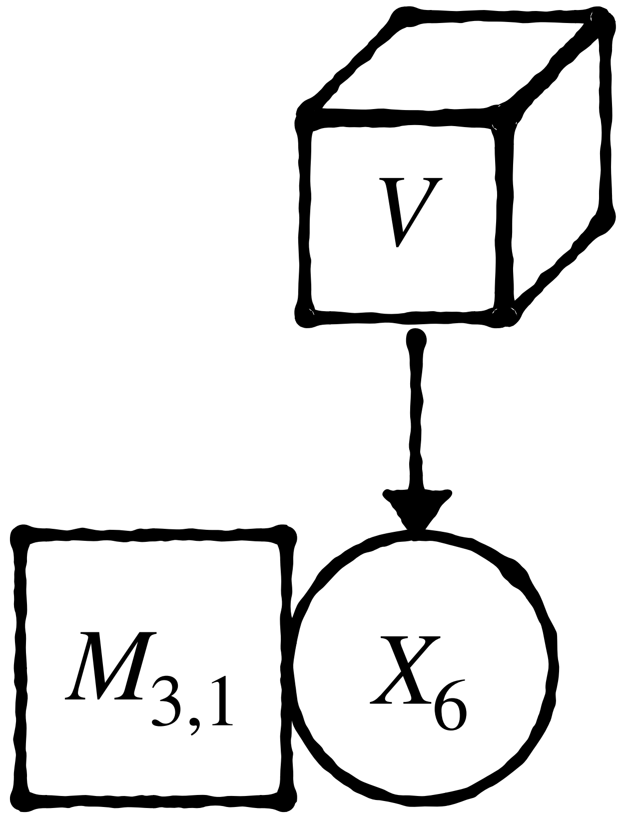

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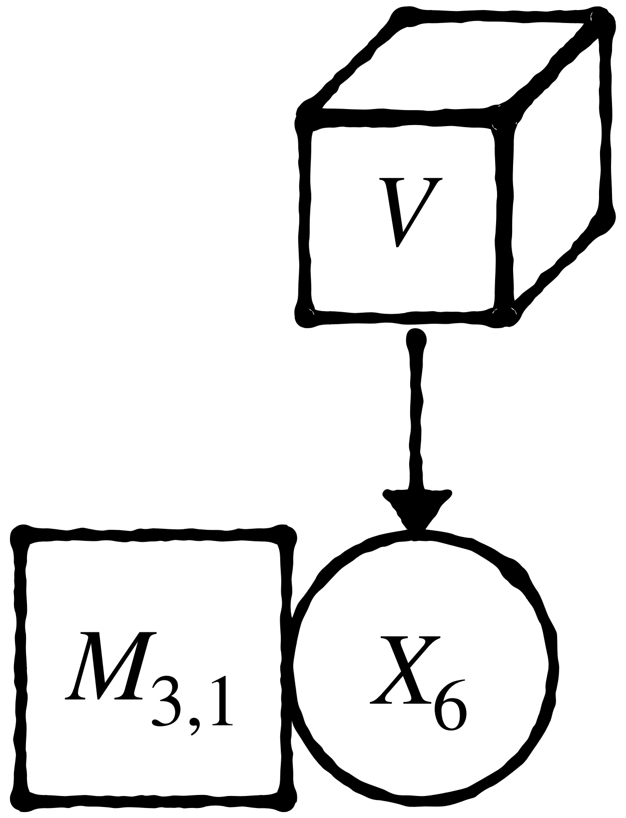

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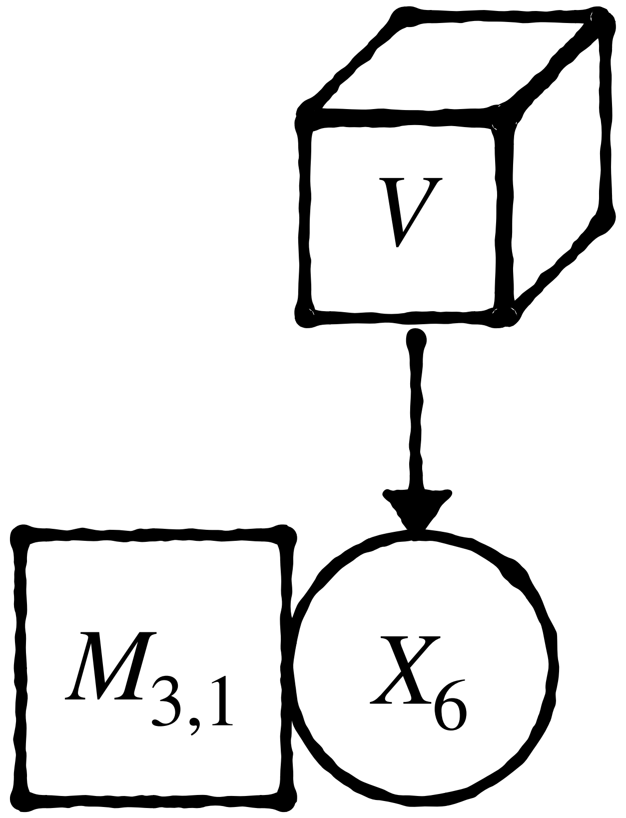

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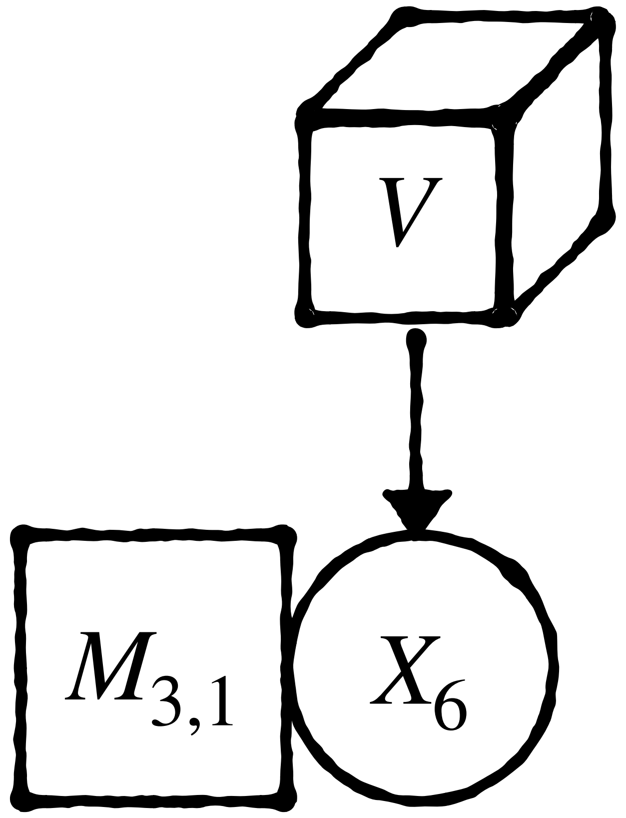

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Physical Yukawa couplings come from holomorphic Yukawa couplings

$$\lambda_{IJK} \sim \int_X \nu_I \wedge \nu_J \wedge \nu_K \wedge \Omega$$

scaled by matter field metric

$$K_{IJ} \sim \int_X \nu_I \wedge \star \nu_J$$

Direct perturbative computation with aid of ML techniques possible! [Constantin et al. (2024)]

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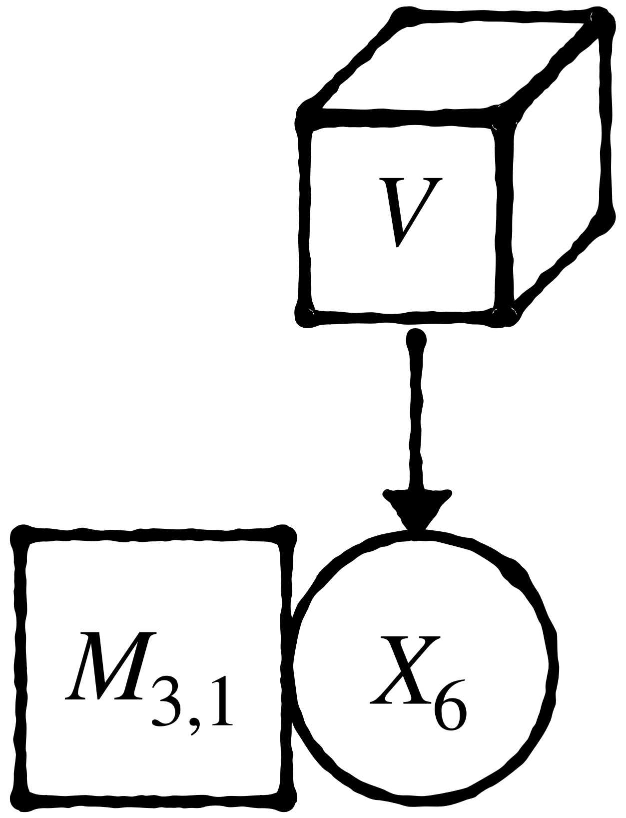

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Coefficient calculation - DIFFICULT:

- K_{IJ} **requires knowledge of metric of** X
- **complicated dependence on moduli fields**
- **non-perturbative corrections are hard**

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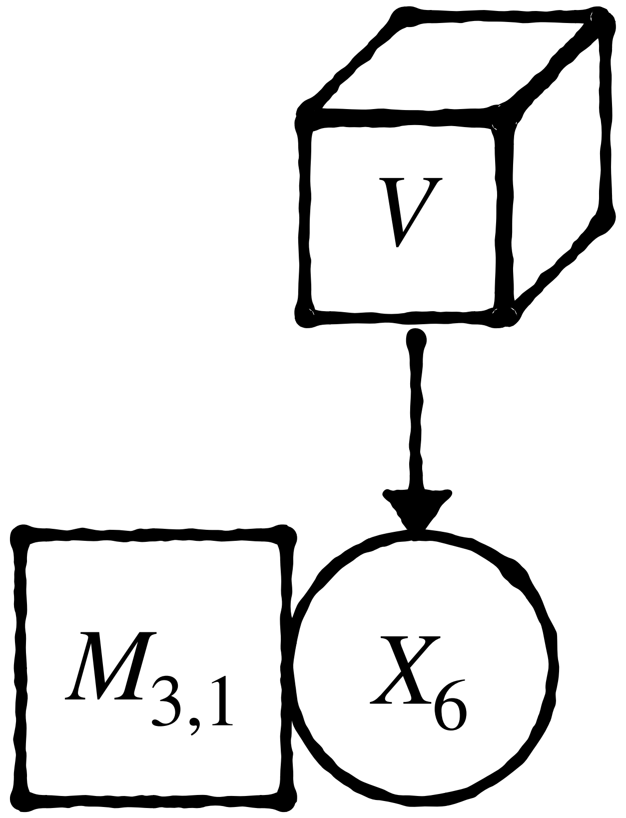

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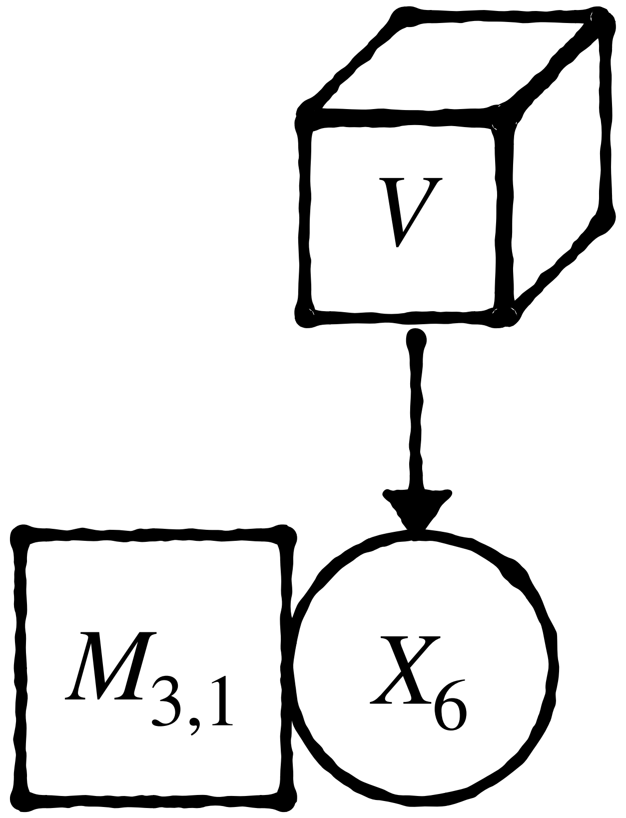

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Step 2 : Wilson-line breaking

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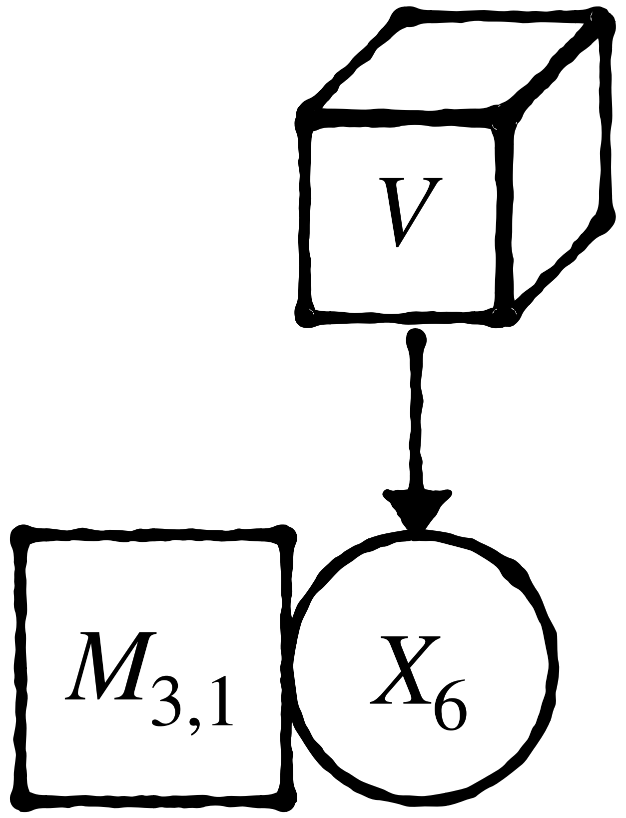

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Select vector bundle + Get $G_{\text{GUT}} \subset E_8$ →

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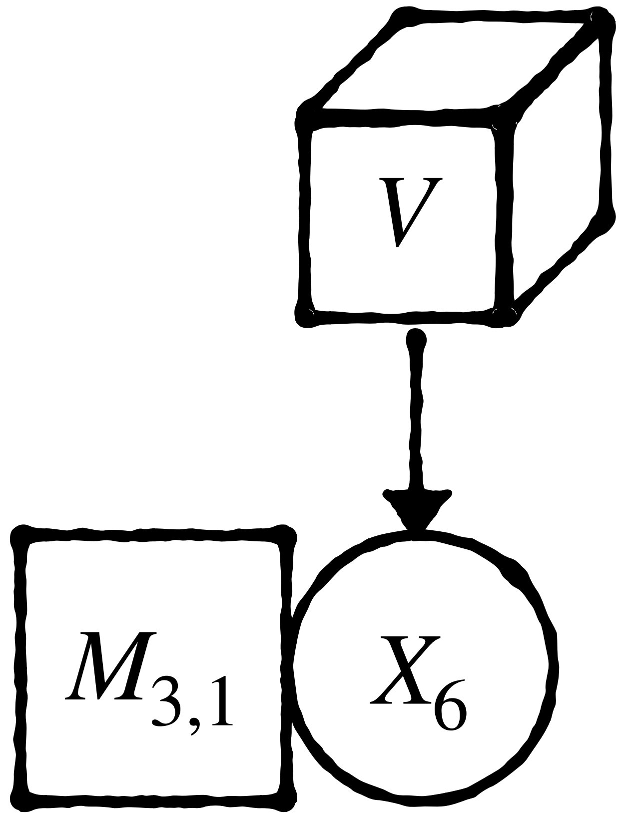

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Select vector bundle + Get $G_{\text{GUT}} \subset E_8$ → **Step 1 : GUT Gauge Group**

spectrum

Break G_{GUT} to G_{SM} → **Step 2 : Wilson-line breaking**

Step 3 : $U(1)$ symmetries

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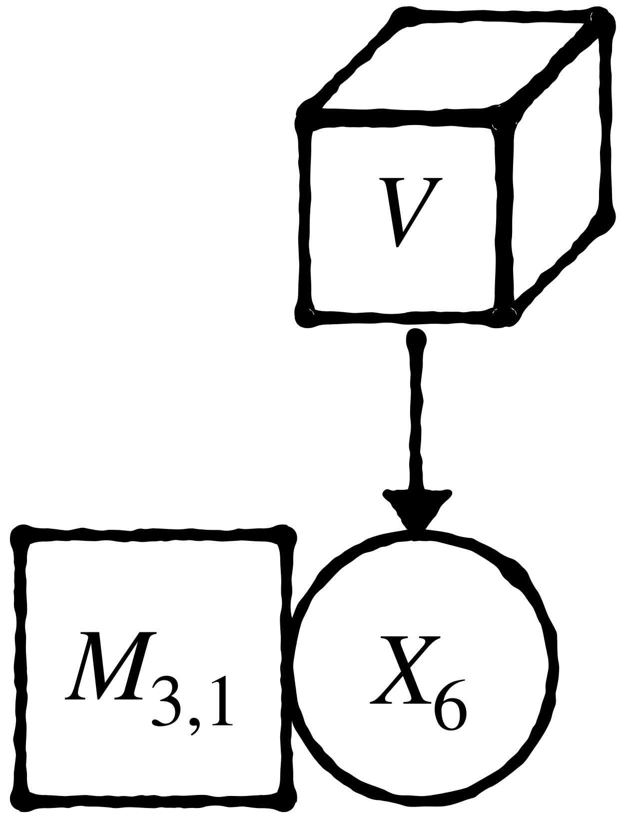

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Break G_{GUT} to G_{SM} → **Step 2 : Wilson-line breaking**

coupling

Special to this string construction! → **Step 3 : $U(1)$ symmetries**

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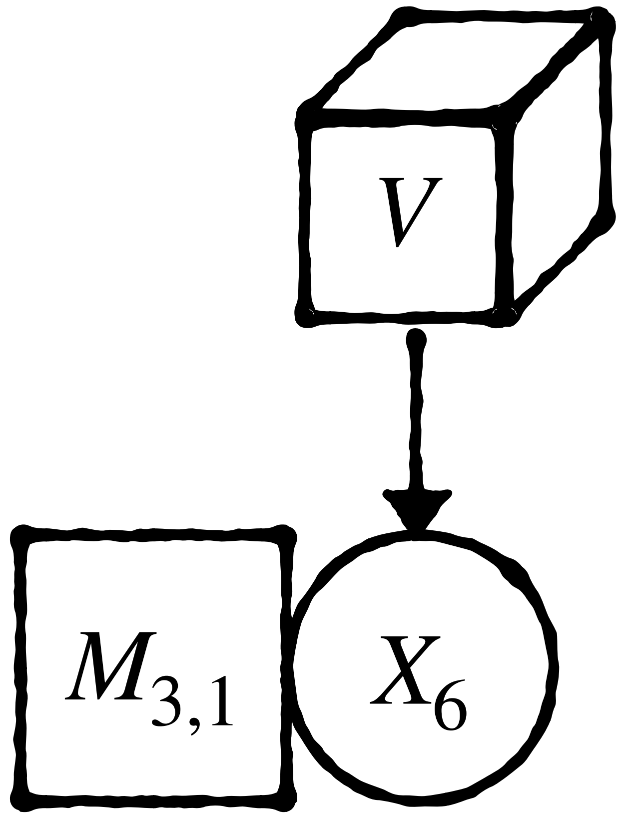

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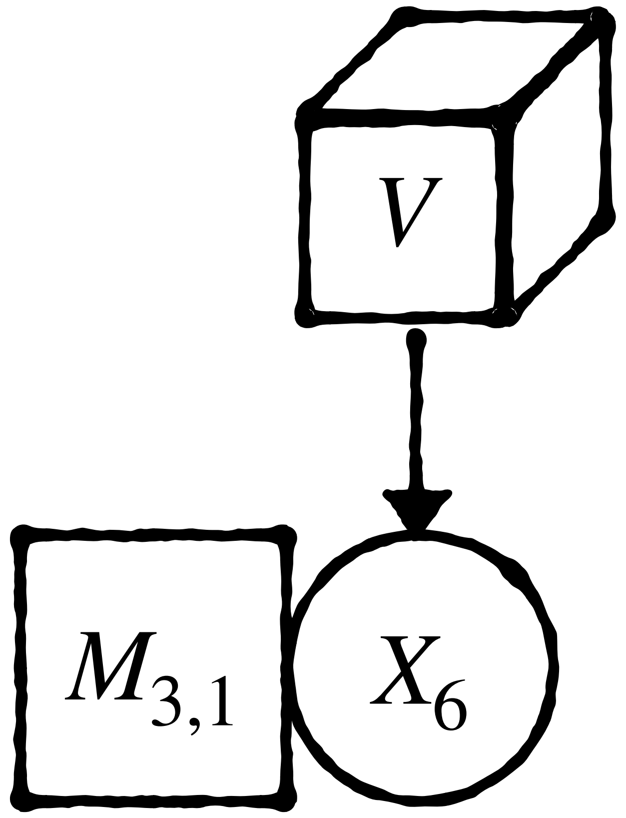

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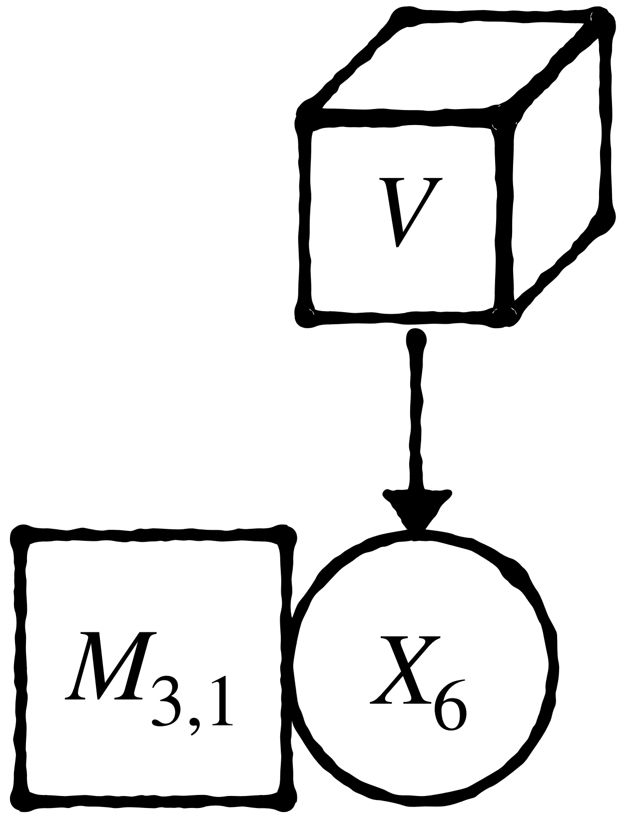

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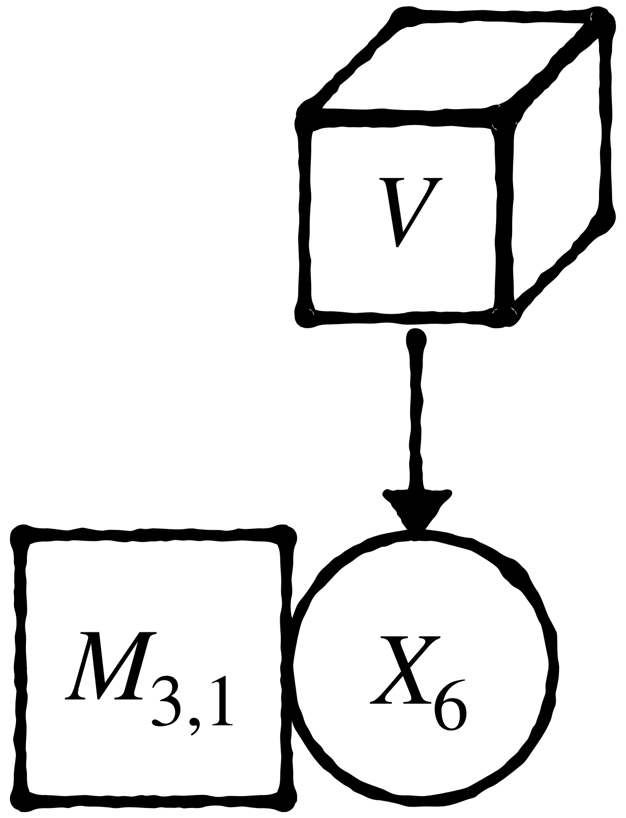

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H_d	$(\mathbf{1},\mathbf{2})_{-3}$	$\bar{\mathbf{5}}_H$	$q = e_a + e_b$	Down-Higgs
H_u	$(\mathbf{1},\mathbf{2})_3$	$\mathbf{5}_H$	$q = -e_a - e_b$	Up-Higgs
ν_A	$(\mathbf{1},\mathbf{1})_1$	$\mathbf{1}$	$q = e_a - e_b$	RH neutrinos
ϕ_A	$(\mathbf{1},\mathbf{1})_1$	$\mathbf{1}$	$q = e_a - e_b$	Moduli singlets
$\Phi_B = e^{-T_B}$	$(\mathbf{1},\mathbf{1})_1$	$\mathbf{1}$	k_a^B	Kähler moduli

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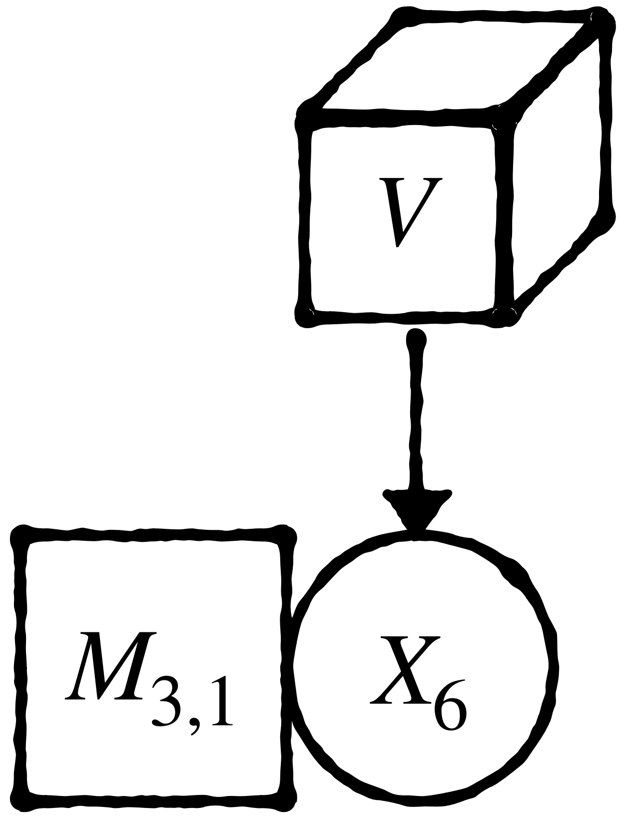

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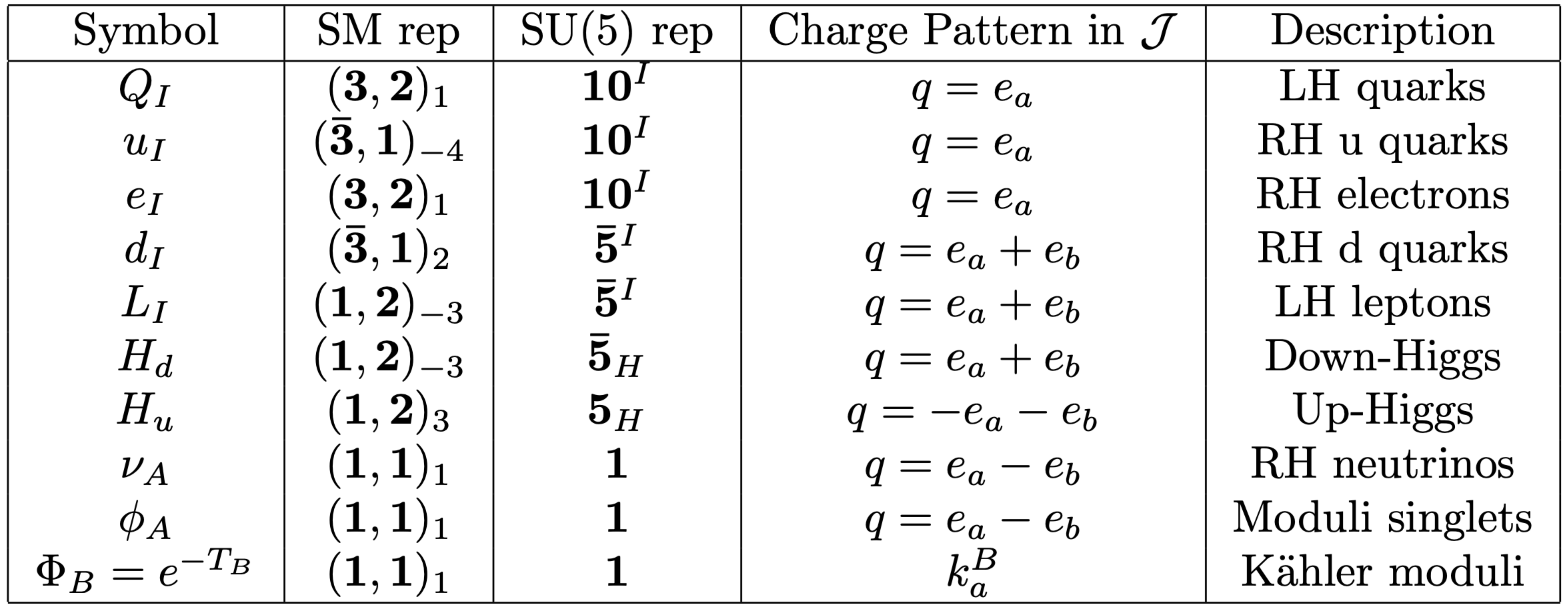

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Low-energy matter field structure: $\mathbf{248}$ of E_8

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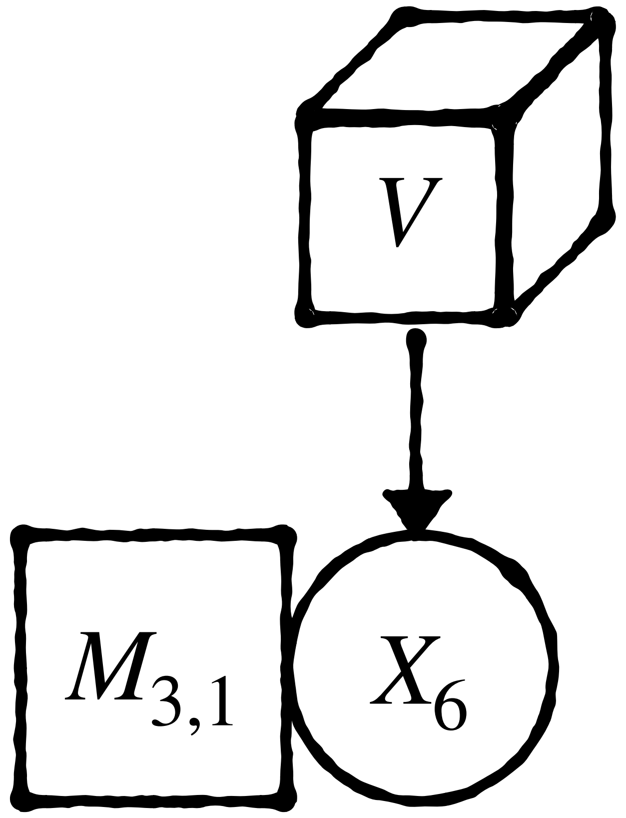

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Kähler moduli: $T_B = t_B + i\chi_B$, transforms as

$$\chi^B \mapsto \chi^B + k_a^B \Lambda^a$$

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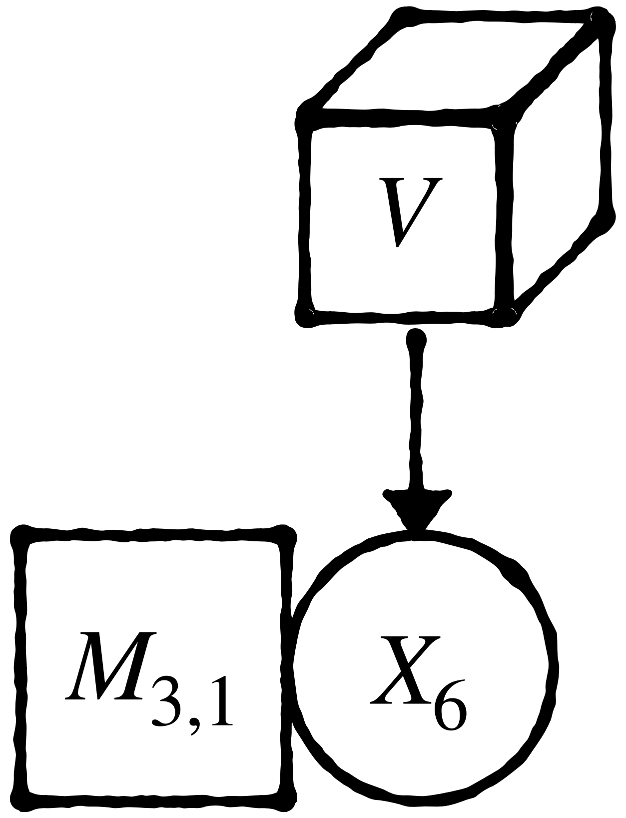

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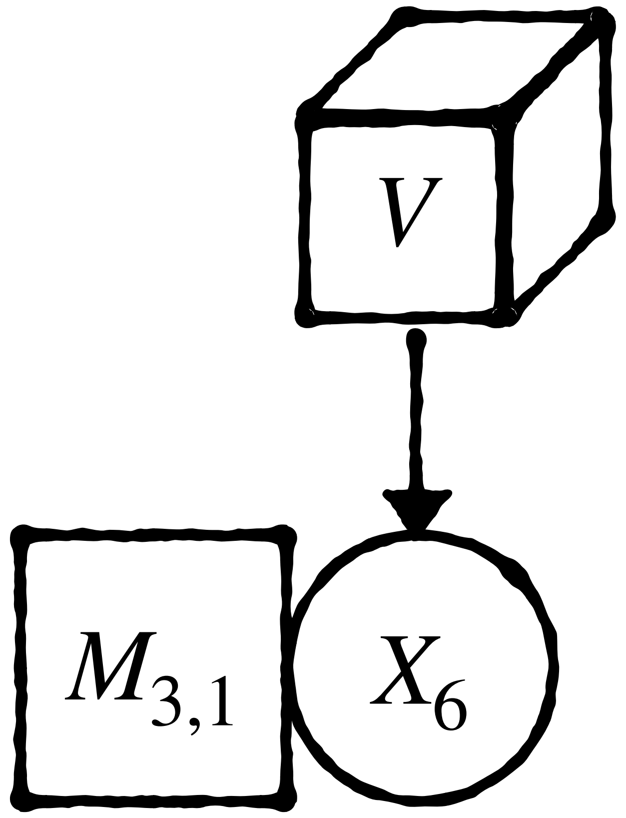


Mathematics	Physics
Topology	Spectrum
Geometry	Couplings

Large number of models with SM spectrum found by computational methods!
[Anderson et al. (2013), Constantin et al. (2018)]

Low-energy matter field structure: $\mathbf{248}$ of E_8

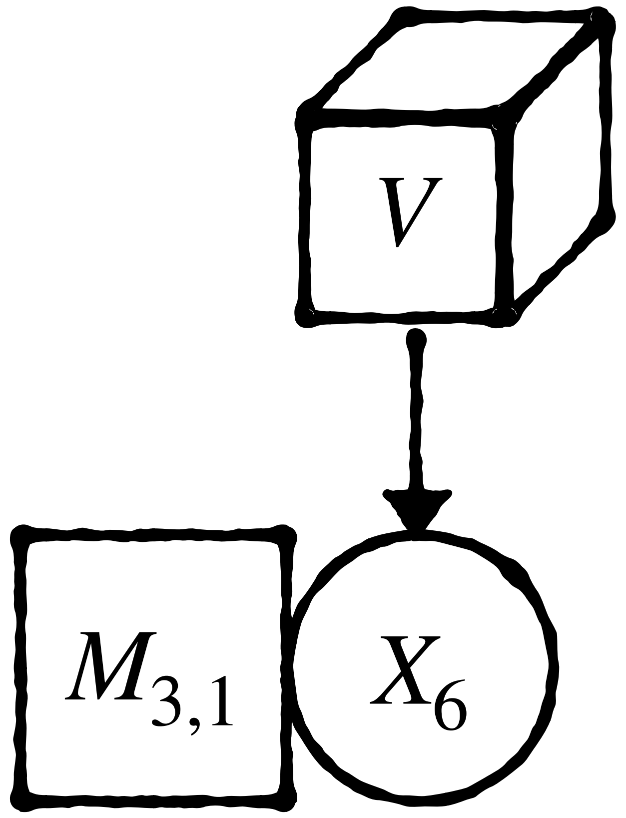
Kähler moduli: $T_B = t_B + i\chi_B$, transforms as

$$\chi^B \mapsto \chi^B + k_a^B \Lambda^a$$

Heterotic CY Compactifications and Line Bundle Standard Models

Ingredients:

- heterotic $E_8 \times E_8$ superstring theory
- Calabi-Yau threefold X
- vector bundle $V \to X$ for vector multiplets



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Step 3 : $U(1)$ symmetries

- Globally remnant $\mathcal{J} = S(U(1)^5)$ symmetries (GS-anomalous in HE) - exploit this to constrain couplings in effective theory
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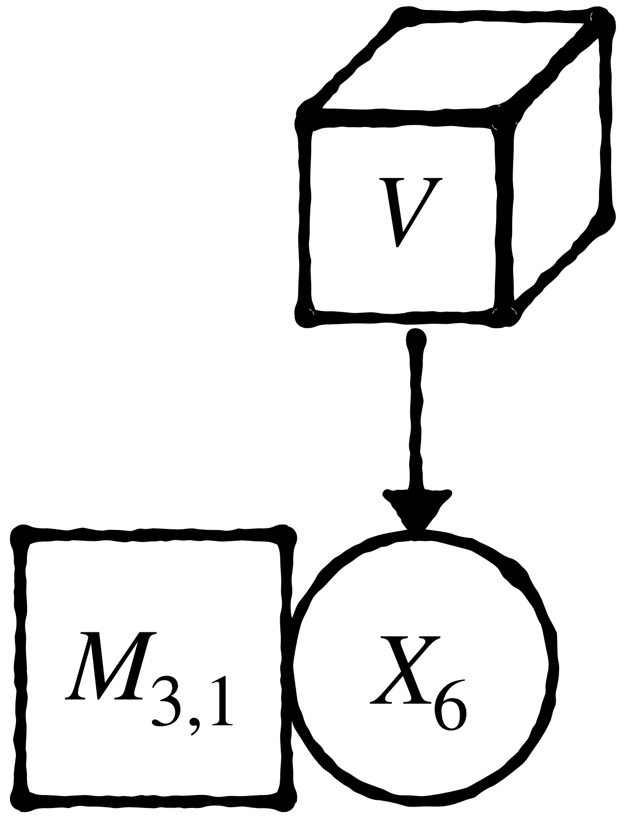

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A field C with charge $\mathbf{q}(C)$ in $\mathscr{J}$ will transform as:

$$C \mapsto e^{-i\mathbf{q}(C)\cdot\epsilon}C$$

Use this to constrain the form of the Yukawa couplings in:

$$W = \hat{\Lambda}^u_{IJ}(\phi)H^u Q^I u^J$$

such that for example

$$\mathbf{q}(\hat{\Lambda}^u_{IJ}(\phi)) = -\mathbf{q}(H^u Q^I u^J)$$

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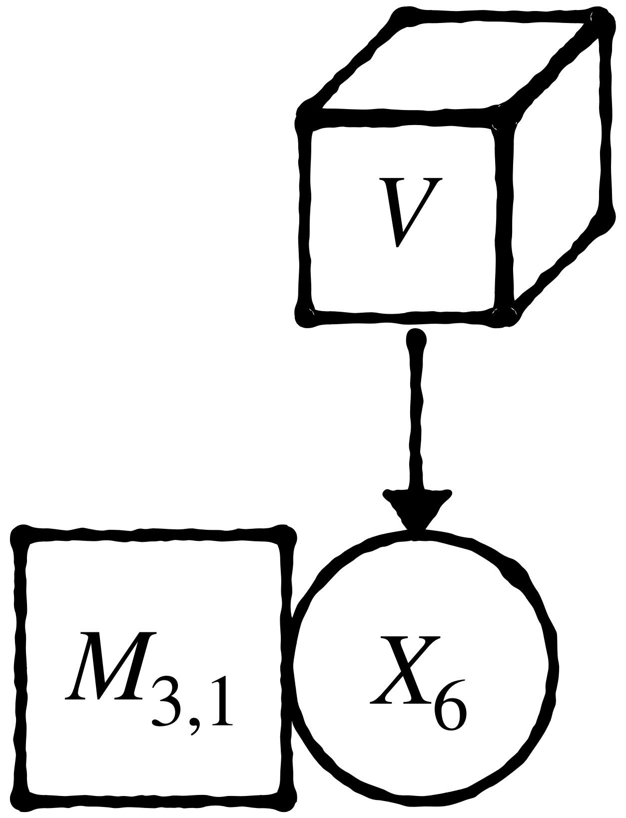

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a function of moduli fields ϕ_a and Φ_a

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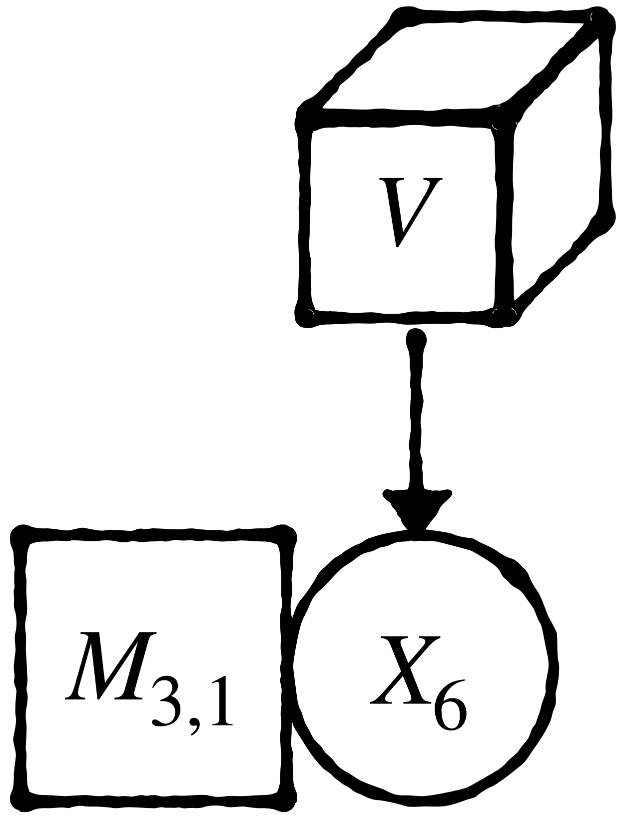

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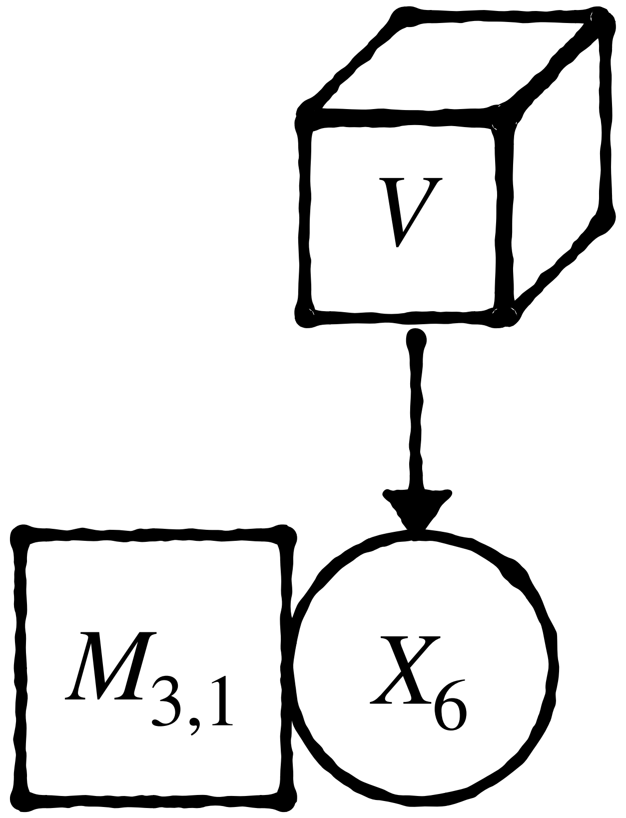

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Can we calculate couplings and extract flavour physics?

Low-energy effective action and symmetries

- In low-energies the models are 4d $\mathcal{N} = 1$ SUSY Standard Models [Anderson et al. (2012)]
- Relevant terms in the Kähler potential and superpotential:

$$K = \hat{k}^u H^u \bar{H}^u + \hat{k}^d H^d \bar{H}^d + K^C_{IJ} C^I \bar{C}^J + \dots$$

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Yukawa couplings

- fermion masses and mixings
- gives masses and mixings of quarks and charged lepton masses

$$m_u,\, m_c,\ \dots,\, V_{\text{CKM}}$$

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- electroweak breaking scale
- this should be suppressed compared to the Planck scale as

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Neutrino Physics

- want to get three light families of neutrinos via the see-saw mechanism
- bundle moduli ϕ_i with zero VEVs act as RH neutrinos
- $\hat{\mu}_I L^I H^u$ - Dirac mass terms
- $\hat{\mu}_0 \supset M_{ij}\phi_i\phi_j$ - Majorana mass terms

Goal

- Using the global $U(1)$ symmetries $\mathscr{J}$ we write down low-energy effective Lagrangian
- Extract relevant phenomenological terms - are there models that give good phenomenology?

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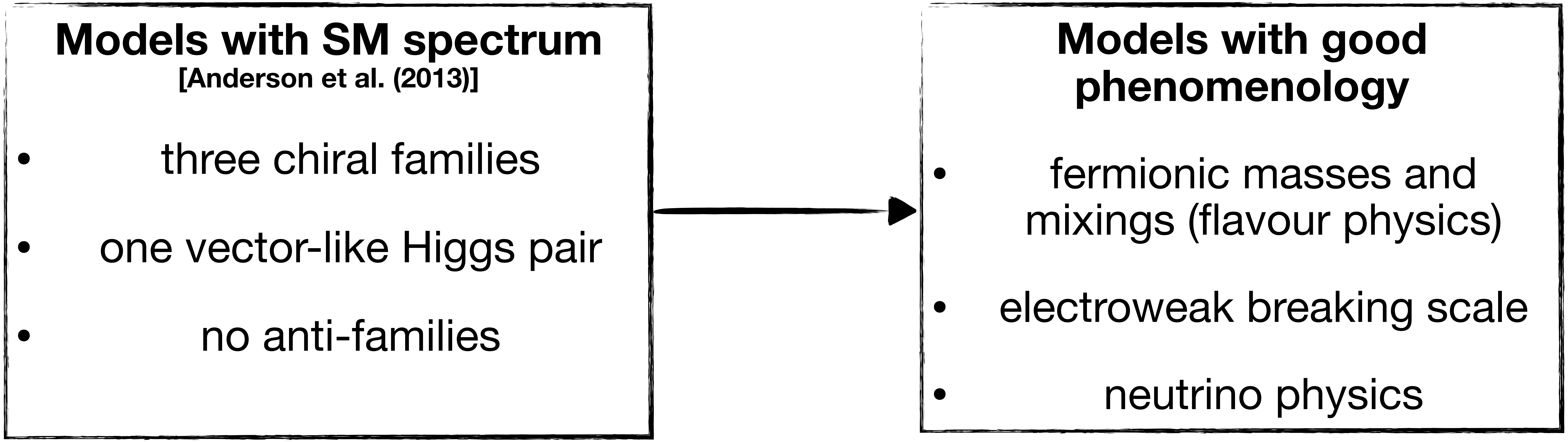

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Models with good phenomenology

- fermionic masses and mixings (flavour physics)
- electroweak breaking scale
- neutrino physics

Method

Models with SM spectrum ⟶ Models with good pheno

Step 1: μ-term suppression

- $M_H \ll M_P$, use VEVs of moduli fields to set small μ-term
- find combinations of VEVs that would set $\mu \to 0$

Step 2: masses and mixings

- for each combination use remaining VEVs to obtain masses and mixings
- gradient descent algorithm to find suitable VEVs and Yukawa coefficients

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- compute Dirac and Majorana couplings and construct neutrino masses - three light families?

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$$Y_u = \begin{pmatrix} \phi_1\phi_3\phi_4 & \Phi_2 & \phi_3\phi_4 \\ \Phi_2 & \phi_1^2 & \phi_3 \\ \phi_3\phi_4 & \phi_3 & 1+\phi_2\phi_3 \end{pmatrix}$$

Descent algorithm on

a_{ij}, $\langle\phi_a\rangle$, $\langle\Phi_i\rangle$

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check if VEV choices consistent with possible neutrino physics

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Results of Scans

General Observations

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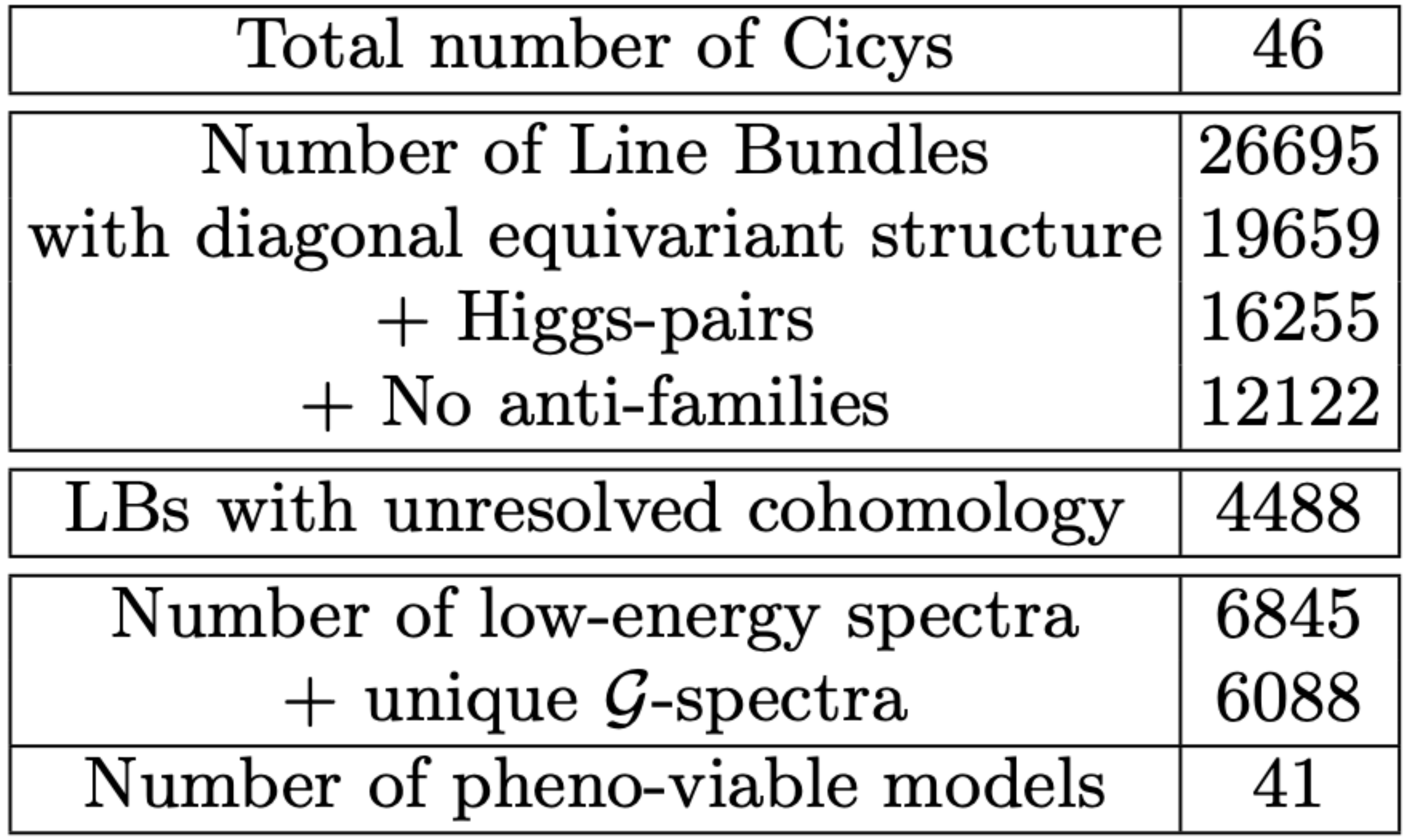
Numbers



Total number of Cicys	46
Number of Line Bundles	26695
with diagonal equivariant structure	19659
+ Higgs-pairs	16255
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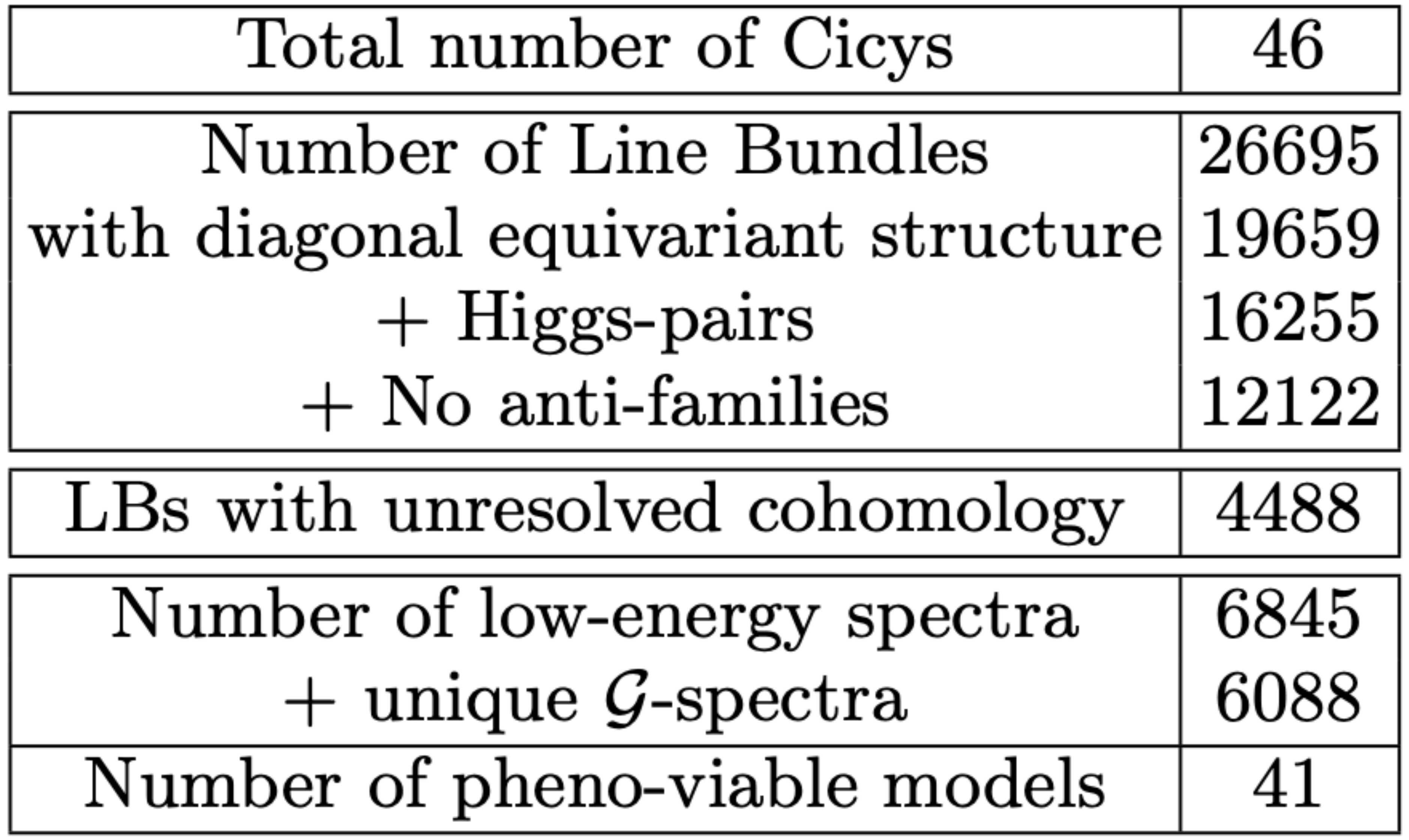

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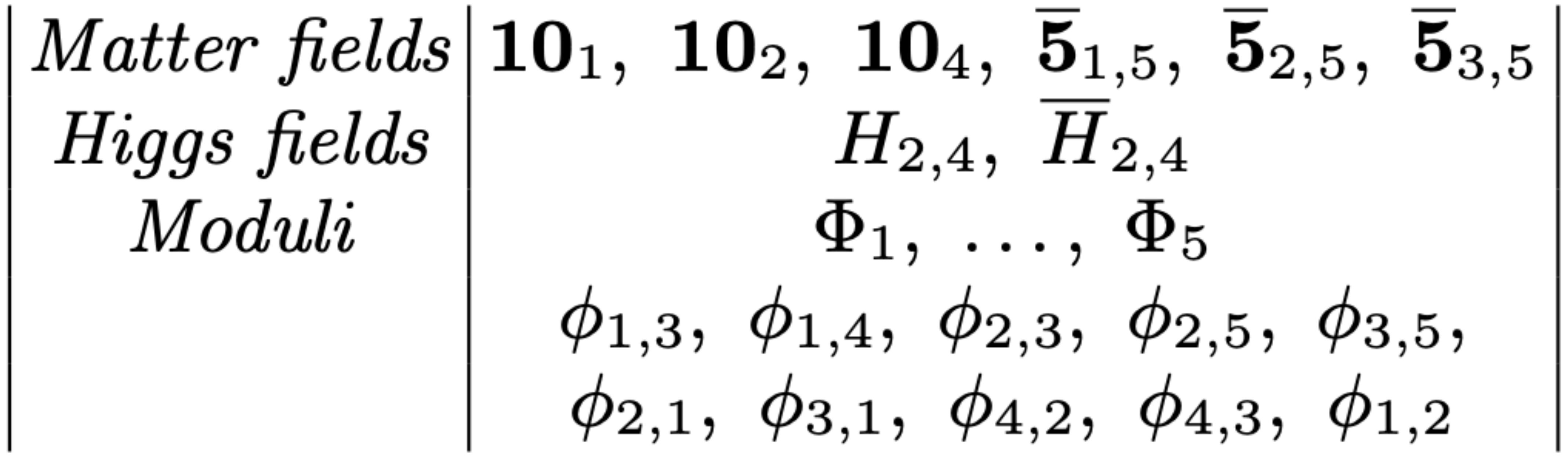
Phenomenological Observations

- Order-one coefficients may generate hierarchy - their ranges are bounded
- Need full-rank up-Yukawa textures
- Trade-off between increasing rank of down-Yukawa and decreasing order-one range
- Generically R-parity violating terms not suppressed

Example Model

μ-term analysis

Downstairs Spectrum



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Compute μ-term insertions

$$\begin{aligned} \mu\ \sim\ & \phi_{1,2}\phi_{2,1} + \phi_{1,3}\phi_{3,1} + \phi_{1,2}\phi_{3,1}\phi_{2,3} \\ & + \phi_{4,3}\phi_{3,1}\phi_{1,4} + \phi_{4,2}\phi_{2,1}\phi_{1,4} + \phi_{1,2}^2\phi_{2,1}^2 \\ & + \cdots\,, \end{aligned}$$

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μ-term can be consistently set to zero if we pick

$$\left\{\langle\Phi_3\rangle, \langle\Phi_4\rangle, \langle\Phi_5\rangle, \langle\phi_{1,3}\rangle, \langle\phi_{1,4}\rangle, \langle\phi_{2,4}\rangle, \langle\phi_{2,5}\rangle, \langle\phi_{3,5}\rangle, \langle\phi_{1,2}\rangle\right\} \to 0$$

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Example Model

Yukawa Masses

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Yukawa Insertions

$$\Lambda^u \sim \begin{pmatrix} \Phi_5 & 0 & \Phi_5\,\phi_{1,4} + \phi_{2,1} \\ 0 & 0 & 1 \\ \Phi_5\,\phi_{1,4} + \phi_{2,1} & 1 & \Phi_5\,\phi_{1,4}^2 + \phi_{1,4}\,\phi_{2,1} \end{pmatrix}$$

$$\Lambda^d \sim \begin{pmatrix} \Phi_2\,\Phi_5\,\phi_{2,1} + \phi_{3,1} & \Phi_2\,\Phi_5 & 1 \\ \Phi_2\,\Phi_5 & 0 & 0 \\ \Phi_2\,\phi_{2,1}^2 + \phi_{1,4}\,\phi_{3,1} & \Phi_2\,\Phi_5\,\phi_{1,4} + \Phi_2\,\phi_{2,1} & \phi_{1,4} \end{pmatrix}$$

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Scan of VEV values

$$\Phi_2 \to 0.01,\ \Phi_5 \to 0.0130746,\ \phi_{1,4} \to 0.370977,$$
$$\phi_{2,1} \to 0.47089,\ \phi_{3,1} \to 0.1\,,$$

Example Model

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Masses and mixing

$$(m_u\,, m_d\,, m_t) = (0.0216{,}1.27{,}172.4)\text{GeV}$$
$$(m_d\,, m_s\,, m_b) = (0.00467{,}0.093{,}4.18)\text{GeV}$$
$$(m_e\,, m_\mu\,, m_\tau) = (0.000511{,}0.106{,}1.78)\text{GeV}$$

$$V_{\text{CKM}} = \begin{pmatrix} 0.970 & 0.242 & 0.00358 \\ 0.242 & 0.969 & 0.0448 \\ 0.00737 & 0.0444 & 0.999 \end{pmatrix}$$

Short Summary

- Explored possible phenomenological issues and properties of heterotic line bundle Standard Models up to Picard number 5

- $\mathcal{O}(10)$ models with suppressed μ-term and accurate charged fermion masses and mixings can be obtained

- Neutrino physics - nothing interesting so far…

- And R-parity needs more work too…

Moduli Stabilisation in String Theory

- So far we have only set moduli VEVs to the values that we need to get phenomenological agreements.
- Any realistic attempt to reproduce string models must address the large number of light scalar moduli fields.

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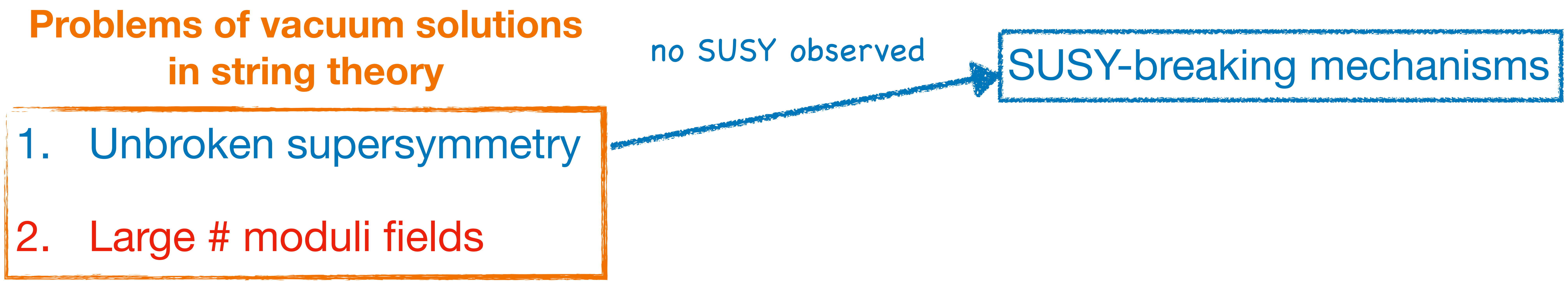

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no SUSY observed

SUSY-breaking mechanisms

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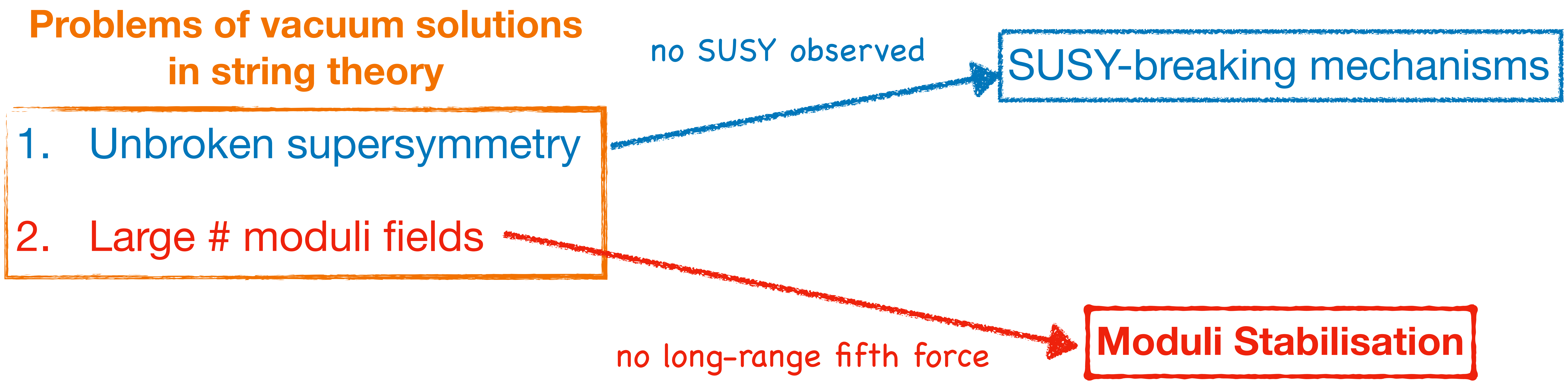

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no long-range fifth force

Moduli Stabilisation

Moduli Stabilisation in String Theory

Types of Moduli Fields

- complex structure moduli z_I
- Kähler moduli T_i
- gauge bundle moduli ϕ_a
- axiodilaton τ

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Non-vacuum solutions

- p-form quantised fluxes
- localised sources (D-branes)

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Flux Compactifications

- sources of stress-energy in internal space - non-vacuum solutions
- small corrections to vacuum solution
- lead to SUSY-breaking mass splitting
- generate scalar potential V for moduli fields - masses

Moduli Stabilisation in String Theory

Construction of isolated vacua

Step 1: 4d $\mathcal{N} = 1$ SUSY action

- for a particular type of theory write down structure of superpotential W and Kähler potential K
- Superpotential:

$$W = W_{\text{flux}}(\tau, z_I) + W_{\text{np}}(\tau, z_I, T_i)$$

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$$K = K_{\text{tree}} + \dots$$

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Step 2: Construct V and exploit structure

- key quantity is F-term scalar potential

$$V_F = e^K \left[K^{M\bar{N}} D_M W \bar{D}_{\bar{N}} \bar{W} - 3 |W|^2 \right]$$

- to obtain SUSY vacua - exploit structure such that F-terms of all moduli vanish exactly

$$D_{T_i} W = D_{z_I} W = D_\tau W = 0$$

need quantum corrections in W_{np}, K_{pert}, K_{np} to stabilise T_i

Moduli Stabilisation in Heterotic Theories

The KKLT Scenario

Moduli Stabilisation in Heterotic Theories

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- Start with type IIB flux compactifications on O3/O7 orientifolds
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$$K = -2\log(2^{3/2}g_s^{-3/2}\mathscr{V}) - \log\left(-i(\tau - \bar{\tau})\right) - \log\left(-i\int \Omega \wedge \Omega\right)\dots$$

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$$K = -2\log(2^{3/2}g_s^{-3/2}\overbrace{\mathcal{V}}^{K_{\text{tree}}}) - \log\big(-i(\overbrace{\tau}^{K_{(\alpha')^3}} - \bar{\tau})\big) - \log\left(\overbrace{-i}^{K_{\text{cs}}}\int\Omega\wedge\Omega\right)\dots$$

Key idea 1

if $W_0 = \langle W_{\text{flux}}\rangle$ exponentially small, small SUSY-breaking of fluxes is compensated by W_{np}

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- vacuum occurs in regime where all approximations are valid ($g_s \ll 1$, α'-exp., back-reactions)
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Key idea 2

compactification near conifold singularity allows controllable breaking of SUSY

Moduli Stabilisation in Heterotic Theories

Basics of Periods and Fluxes

Question: What is W_{flux}?

Classical Gukov-Vafa-Witten flux superpotential:

$$W_{\text{flux}} \sim \int_{X_6} G_3 \wedge \Omega$$

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$H_3 = dB_2$

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Define period vector Π as

$$\Pi = \begin{pmatrix} \int \Omega \wedge \beta_A \\ \int \Omega \wedge \alpha^A \end{pmatrix} = \begin{pmatrix} \mathcal{F}_A \\ \mathcal{Z}^A \end{pmatrix},$$

such that

$\mathcal{Z}^A$ - homogeneous projective coords on cs moduli space

$\mathcal{F}$ - prepotential, $\mathcal{F}_B = \dfrac{\partial \mathcal{F}}{\partial \mathcal{Z}^B}$

$\Omega = \mathcal{Z}^A \alpha_A - \mathcal{F}_B \beta^B$ holomorphic 3-form

Moduli Stabilisation in Heterotic Theories

IIB (KKLT) vs Heterotic

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Central Question: Can we do something similar in heterotic $E_8 \times E_8$ theories?

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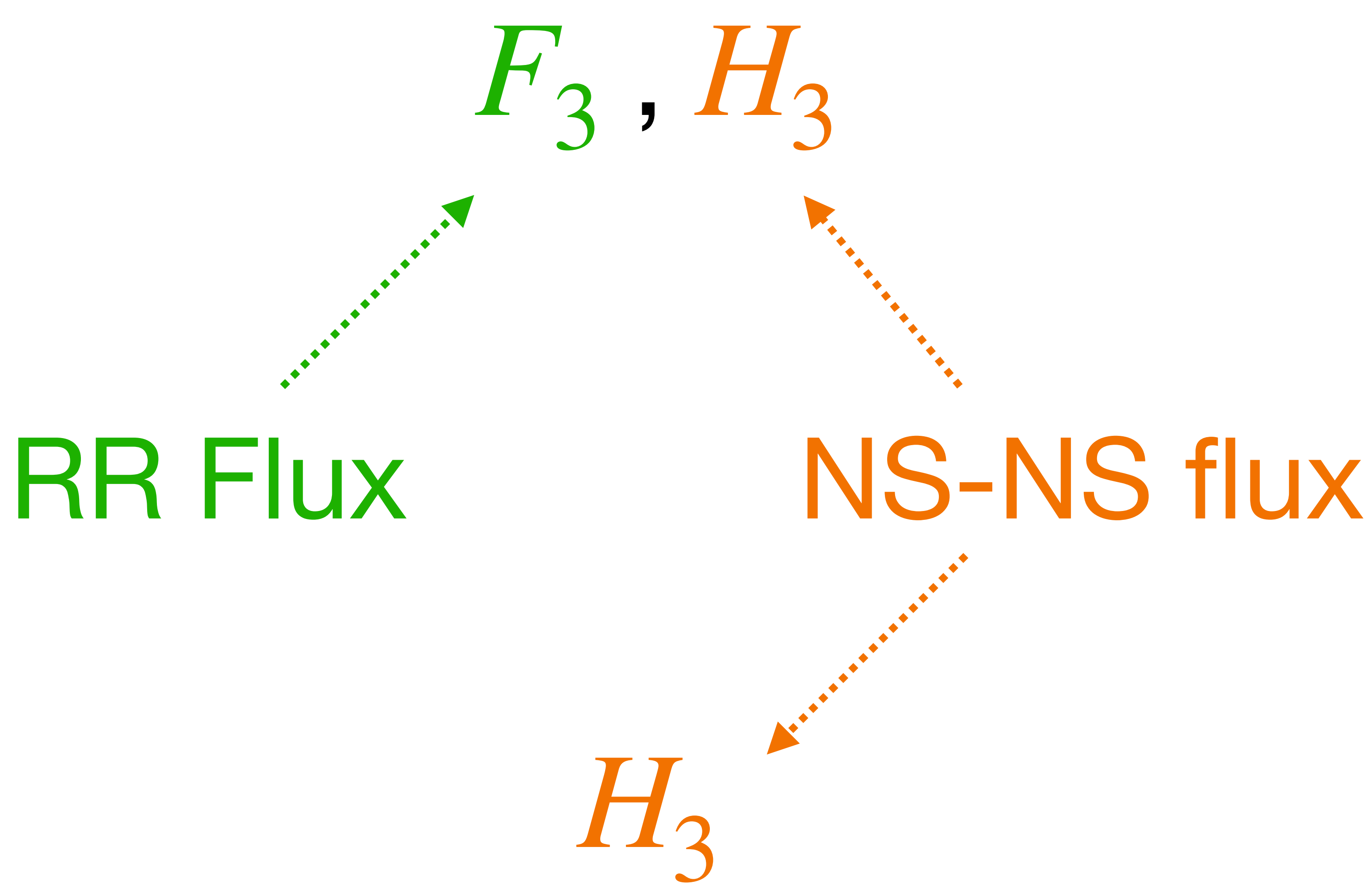

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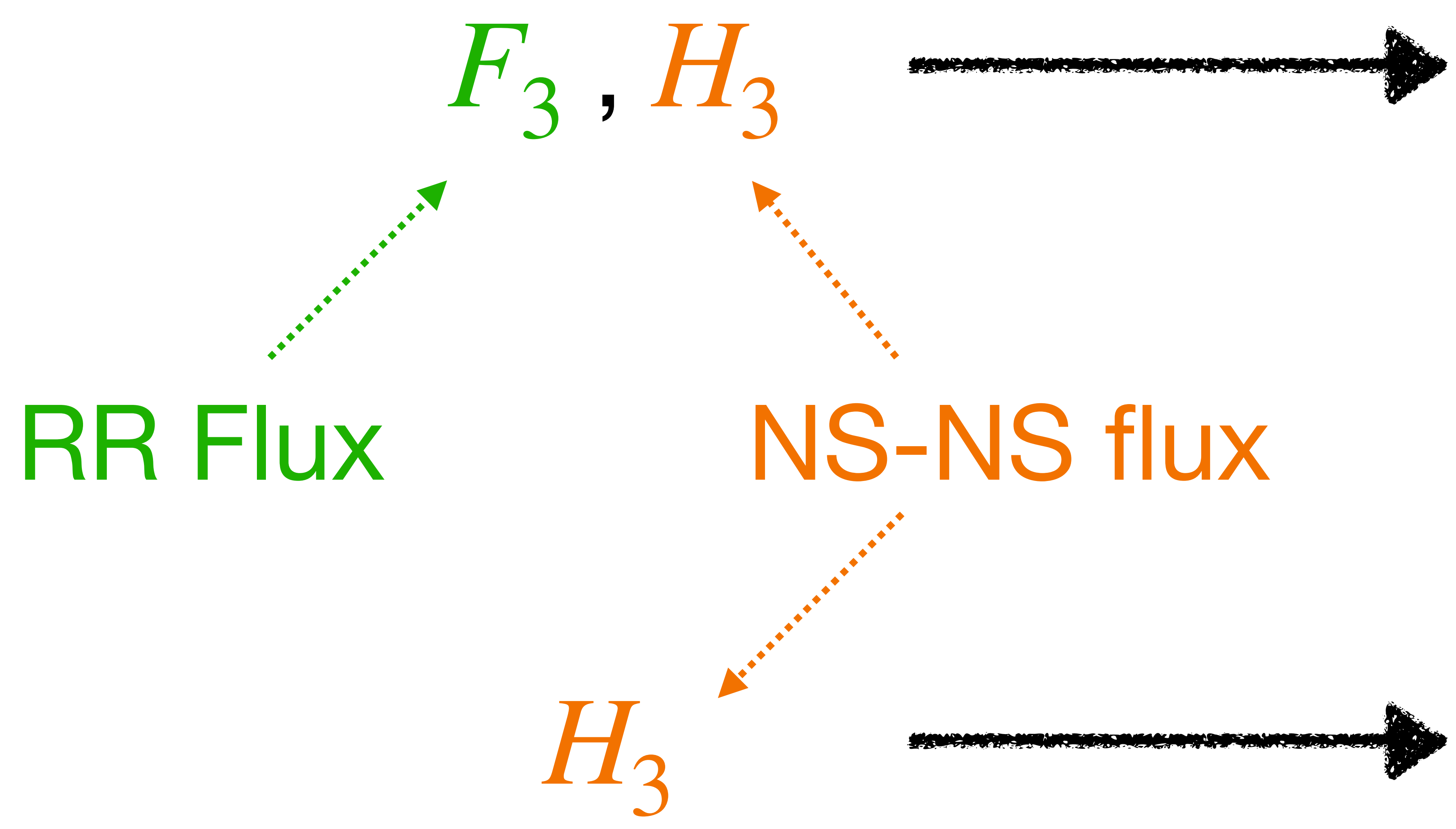

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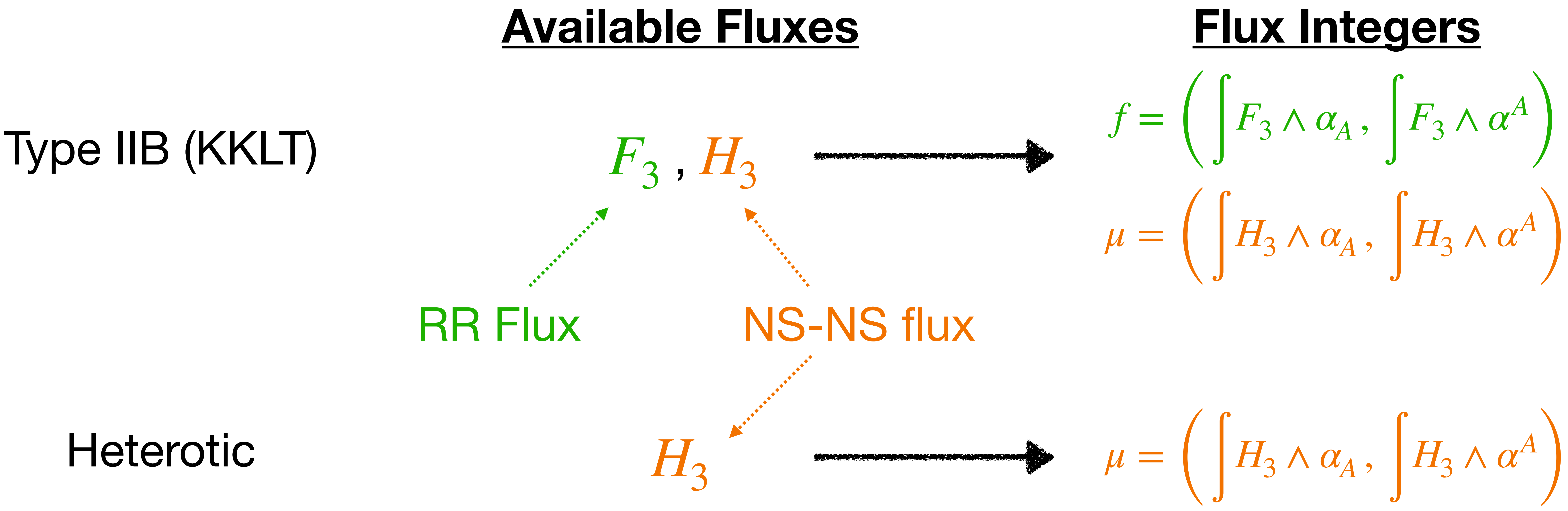

Available Fluxes

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Type IIB (KKLT)

$F_3\,,\,H_3 \longrightarrow$

$$f = \left(\int F_3 \wedge \alpha_A \,, \int F_3 \wedge \alpha^A \right)$$

$$\mu = \left(\int H_3 \wedge \alpha_A \,, \int H_3 \wedge \alpha^A \right)$$

RR Flux

NS-NS flux

Heterotic

$H_3 \longrightarrow$

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some Pfaffian

$$W = W_0 + \mathscr{B}(\tau, z_I, \phi_a) e^{-2\pi T_i} + \dots$$

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No RR fluxes - traditional small W_0 argument does not work - WHAT DO WE DO?

Moduli Stabilisation in Heterotic Theories

Quick Recap

- Natural context to study moduli stabilisation - **flux compactifications**
- Most prominent example - **KKLT scenario**, requires small W_0
- Argument requires **large number of RR fluxes** which does not exist in heterotic theories
- Is it possible that we have W_0 **`accidentally'** small to compete with the small non-perturbative term? For example - some natural region in complex structure moduli space that gives small W_0?

Heterotic Moduli Stabilisation

Setting the scene

Recall introduced symplectic basis of $\alpha_A\,,\beta^A \in H_3(X_6,\mathbb{Z}),\,A=0,\ldots,h^{2,1}$

Defined period vector Π as

$$\Pi = \begin{pmatrix} \int \Omega \wedge \beta_A \\ \int \Omega \wedge \alpha^A \end{pmatrix} = \begin{pmatrix} \mathcal{F}_A \\ \mathcal{Z}^A \end{pmatrix},$$

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- Goal: To analyse whether W_0 can be small in the heterotic setting - only look at complex structure moduli dependence.
- The superpotential and Kähler potential at tree-level are:

$$W = w \int_{X_6} H_3 \wedge \Omega = w \left[n_A \mathcal{Z}^A - m^A \mathcal{F}_A \right] = w \mu^T \eta \Pi$$

$$K = -\log \left(i \int_{X_6} \Omega \wedge \bar{\Omega} \right) = -\log \left[i \left(\bar{\mathcal{Z}}^A \mathcal{F}_A - \mathcal{Z}^A \bar{\mathcal{F}}_A \right) \right] = \log \left(-i \Pi^\dagger \eta \Pi \right)$$

where $\eta = \begin{pmatrix} 0_n & 1_n \\ -1_n & 0_n \end{pmatrix}$ is the symplectic defining matrix.

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Things to calculate

- Scalar potential:

$$V = e^K \left(K^{a\bar{b}} D_a W D_{\bar{b}} \bar{W} - 3\,|\,W|^2\right)$$

- Global SUSY vacua:

$$\frac{\partial W}{\partial Z^a} = 0$$

- Local SUSY vacua:

$$F_a = W_a + K_a W = 0$$

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- The superpotential and Kähler potential at tree-level are:

$$W = w\int_{X_6} H_3 \wedge \Omega = w\left[n_A \mathcal{Z}^A - m^A \mathcal{F}_A\right] = w\mu^T \eta \Pi$$

$$K = -\log\left(i\int_{X_6} \Omega \wedge \bar{\Omega}\right) = -\log\left[i\left(\bar{\mathcal{Z}}^A \mathcal{F}_A - \mathcal{Z}^A \bar{\mathcal{F}}_A\right)\right] = \log\left(-i\Pi^\dagger \eta \Pi\right)$$

where $\eta = \begin{pmatrix} 0_n & 1_n \\ -1_n & 0_n \end{pmatrix}$ is the symplectic defining matrix.

have defined affine versions of the cs coordinate: $Z^A = \dfrac{\mathcal{Z}^A}{\mathcal{Z}^0}$ on the patch $\mathcal{Z}^0 \neq 0$

Recall introduced symplectic basis of $\alpha_A\,, \beta^A \in H_3(X_6, \mathbb{Z}), A = 0,\ldots, h^{2,1}$

Defined period vector Π as

$$\Pi = \begin{pmatrix} \int \Omega \wedge \beta_A \\ \int \Omega \wedge \alpha^A \end{pmatrix} = \begin{pmatrix} \mathcal{F}_A \\ \mathcal{Z}^A \end{pmatrix},$$

Things to calculate

- Scalar potential:

$$V = e^K\left(K^{a\bar{b}} D_a W D_{\bar{b}} \bar{W} - 3\,|\,W|^2\right)$$

- Global SUSY vacua:

$$\frac{\partial W}{\partial Z^a} = 0$$

- Local SUSY vacua:

$$F_a = W_a + K_a W = 0$$

Heterotic Moduli Stabilisation

Large Complex Structure Limit

- Let us first look at large complex structure limit, where $\mathcal{Z}^A \to \infty$.
- In this case the leading pre-potential is

$$\mathcal{F} = -\frac{1}{6}\frac{\tilde{d}_{abc}\mathcal{Z}^a\mathcal{Z}^b\mathcal{Z}^c}{\mathcal{Z}^0}$$

$\tilde{\kappa} = \tilde{d}_{abc}z^a z^b z^c$ **is mirror volume**

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Global SUSY

- Want to set

$$\frac{\partial W}{\partial Z^a} = w\left[in_a - \tilde{d}_{abc}m^b Z^c + \frac{im^0}{2}\tilde{d}_{abc}Z^b Z^c\right] = 0$$

- Crucial obstacle - solving this leaves

$$\mathfrak{I}(W_0) = w\left[-\frac{m^0}{3}\tilde{\kappa}\right]$$

- This cannot be set small since $\tilde{\kappa} \gg 1$

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Local SUSY

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$$F_a = \frac{\partial W}{\partial Z^a} - 2Z_aW = 0$$

- Crucial obstacle - solving this leaves

$$\mathfrak{I}(W_0) = w\left[\frac{2m^0}{3}\tilde{\kappa}\right]$$

- This cannot be set small since $\tilde{\kappa} \gg 1$

$\tilde{\kappa} = \tilde{d}_{abc}z^az^bz^c$ **is mirror volume**

Heterotic Moduli Stabilisation

General Complex Structure

- For general complex structure we do not have a general form of the pre-potential $\mathscr{F}$.
- Typically complicated - given by **hypergeometric functions**…
- Similar general arguments suggest that there is **no consistent SUSY Minkowski vacua**. But doesn't forbid AdS or dS vacua.
- What if we look at explicit examples to analyse general complex structure moduli spaces?

Mirror Quintic

Manifold

$$p = \sum_{k=1} x_k^5 - 5\psi \prod_{k=1}^{5} x_k$$

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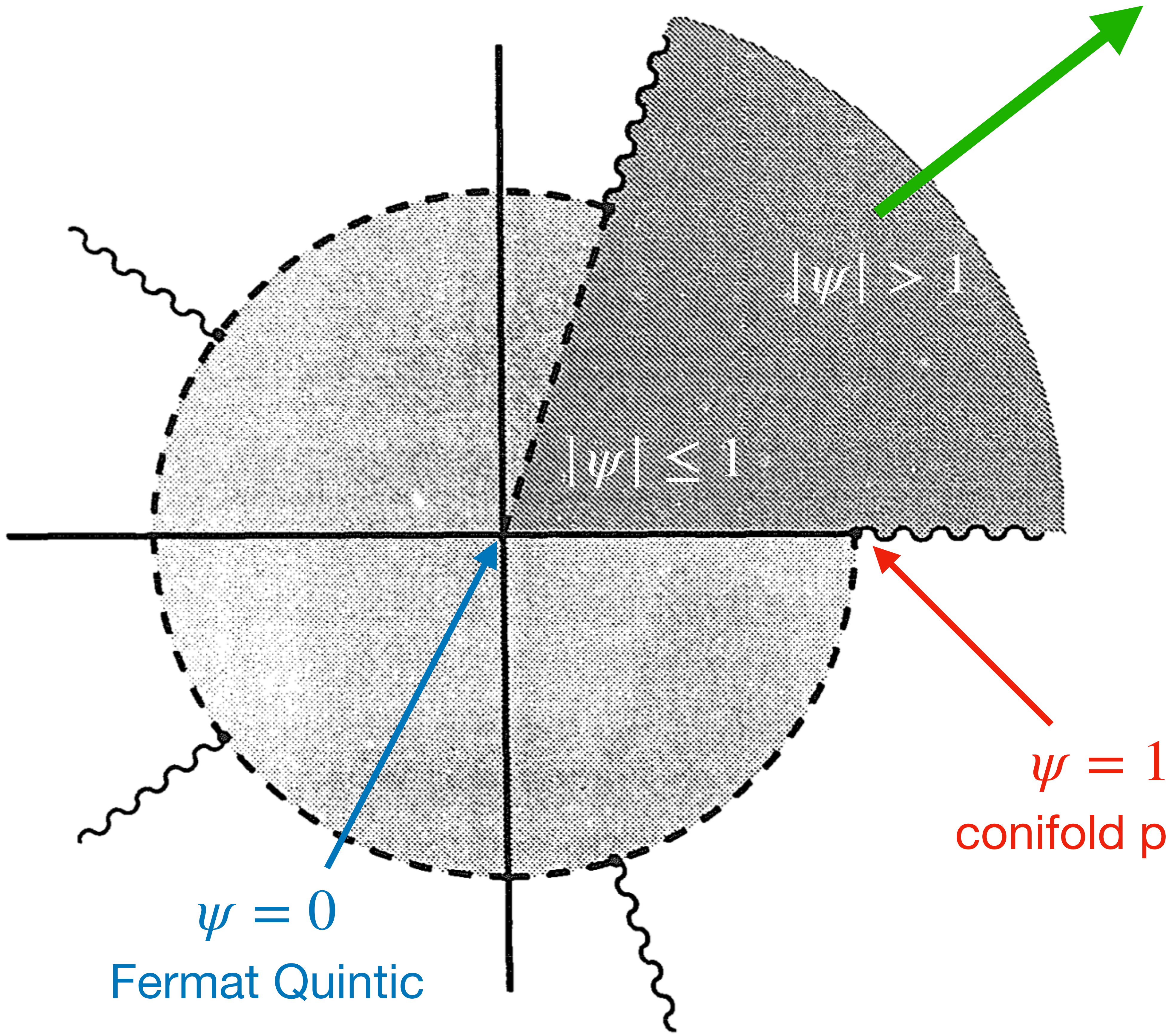
Moduli Space



$\psi \to \infty$
large complex structure

$\psi = 1$
conifold point

$\psi = 0$
Fermat Quintic

Mirror Quintic

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Period Expansion $|\psi| < 1$

$$\varpi_0(\psi) = -\frac{1}{5}\sum_{m=1}^{\infty} \frac{\alpha^{2m}\Gamma(\frac{m}{5})(5\psi)^m}{\Gamma(m)\Gamma^4(1-\frac{m}{5})}$$

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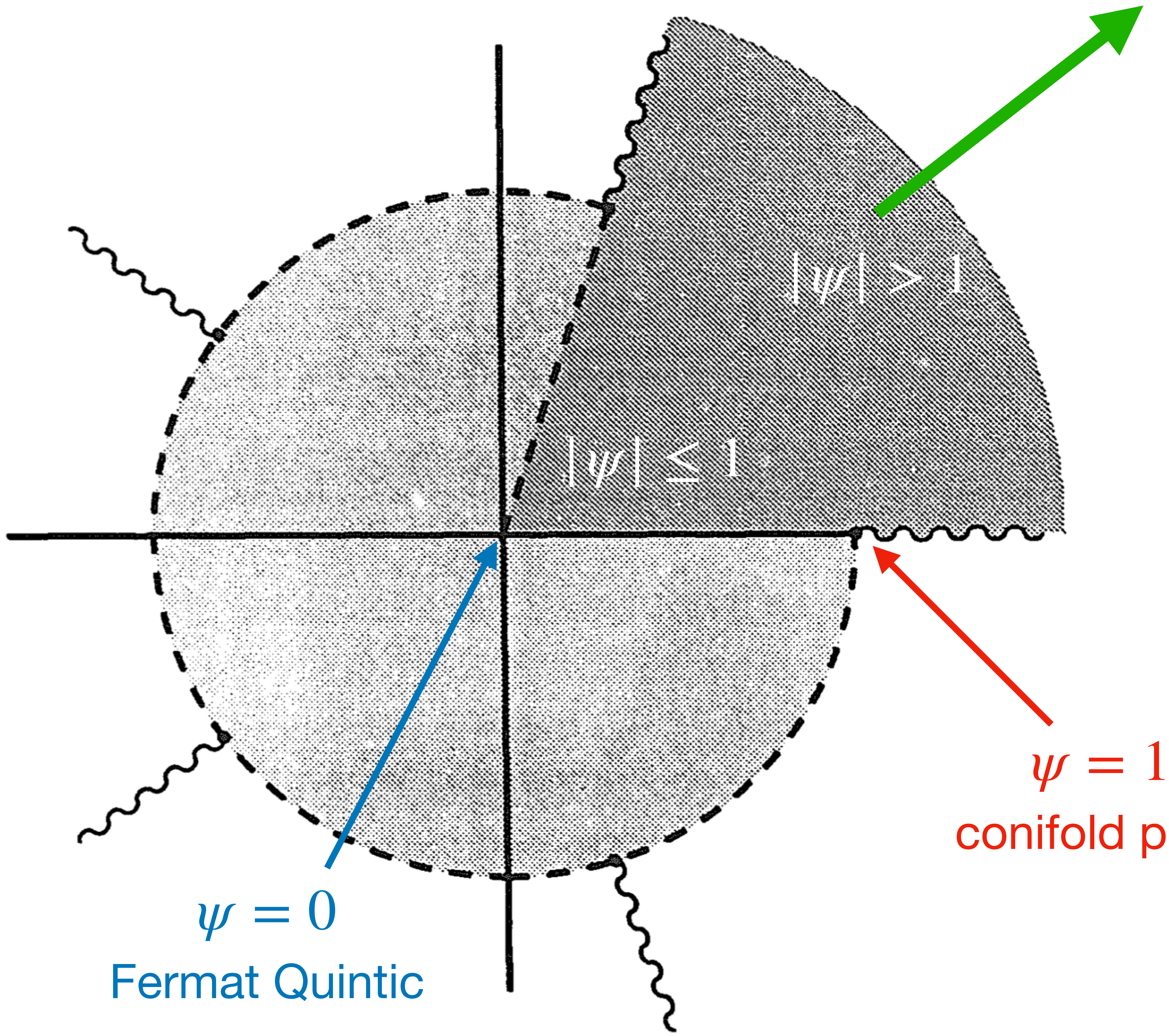

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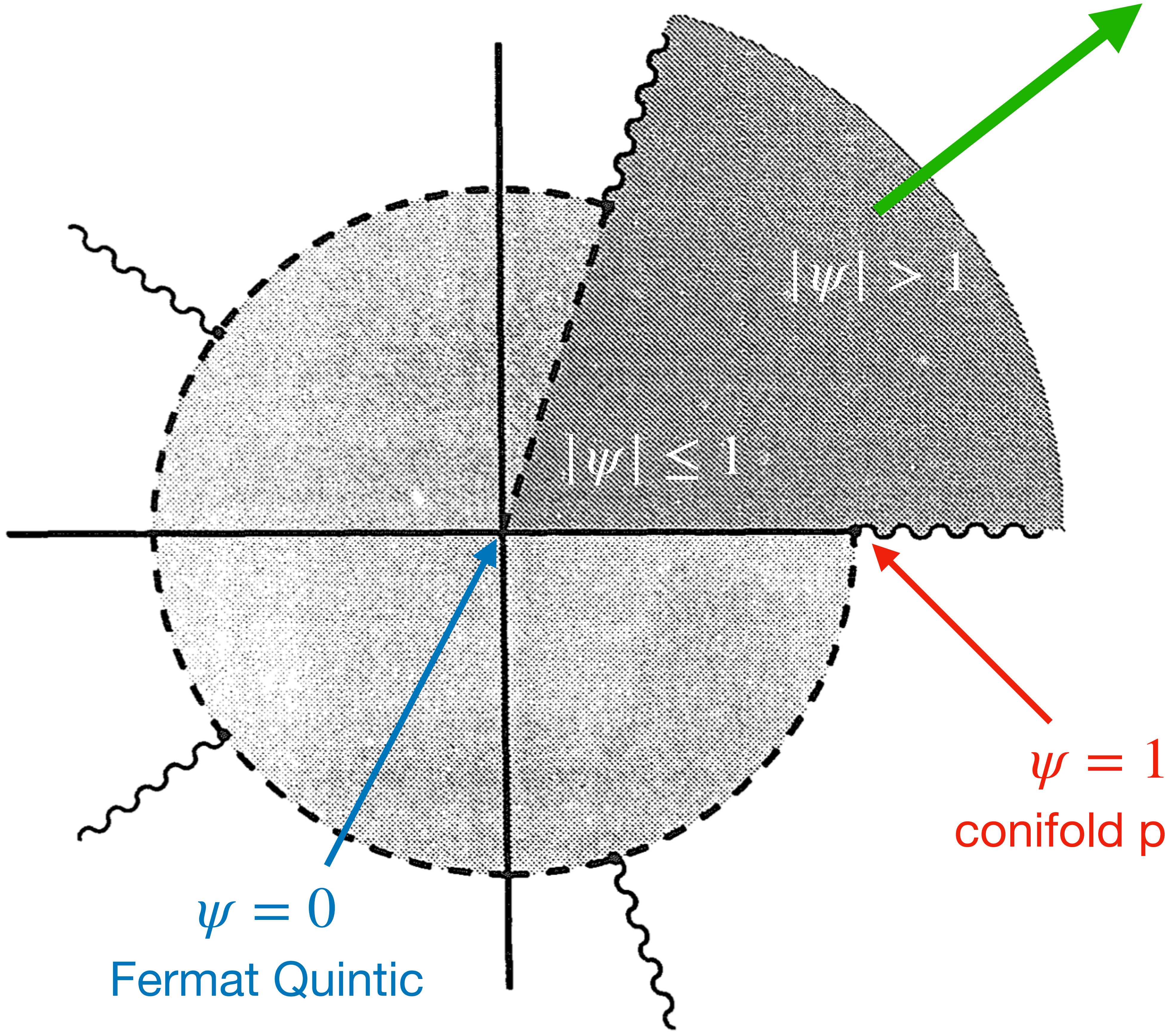
Period Expansion $|\psi| > 1$

$$\varpi_j(\psi) = \sum_{r=0}^{3} \log^r(5\psi) \sum_{n=0}^{\infty} b_{jrn} \frac{(5n)!}{(n!)^5 (5\psi)^{5n}}$$

Moduli Space



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$$\varpi_j(\psi) = -\frac{1}{5}\sum_{m=1}^{\infty}\frac{\alpha^{2m+mj}\Gamma(\frac{m}{5})(5\psi)^m}{\Gamma(m)\Gamma^4(1-\frac{m}{5})} \longrightarrow$$

Period Expansion Basis

$$\varpi(\psi) = \begin{pmatrix} \varpi_2 \\ \varpi_1 \\ \varpi_0 \\ \varpi_{k_1} \end{pmatrix} \longrightarrow$$

Period Vector

$$\Pi = M\varpi$$

Mirror Quintic

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$$\frac{\partial W}{\partial Z^a} = 0$$

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$$F_A = W_A - \frac{\kappa_A}{\kappa}W = 0$$

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Scalar potential

$$V = K^{A\bar{B}}D_AWD_{\bar{B}}\bar{W} - 3|W|^2$$

Mirror Quintic

Global SUSY

- Scan all flux integers $(n_0, n_1, m^0, m^1) \in \{-20\,, \;\ldots\; , 20\}$
- Solve for $\dfrac{\partial \hat{W}}{\partial \psi} = 0$
- Compute values of $|\,\hat{W}/w\,|$
- Best one obtained - $|\,\hat{W}/w\,| \sim 0.064$ at $(n_0, n_1, m^0, m^1) = \pm\,(5, -2, 9, -4)$ and $\psi = -0.55 - 0.25i$.
- $\hat{W}$ not symplectically-invariant under $Sp(4,\mathbb{Z})$!

Mirror Quintic

Local SUSY

- Instead look at quantity

$$\mathscr{V} = e^{K/2}\hat{W}$$

- Repeat search for F-term set to zero,

$$F_a = W_a - \frac{\kappa_a}{\kappa}W$$

- Use solution ψ_0 which obeys $|\psi_0| \leq 1$ to compute $\mathscr{V}$.

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Found Minima

- $(n_0, n_1, m^0, m^1) = (4,0,7,-1)$
- $\psi_{\text{min}} = 0.12 - 0.36i$
- $\mathscr{V}_{\text{min}} \sim 1.25$

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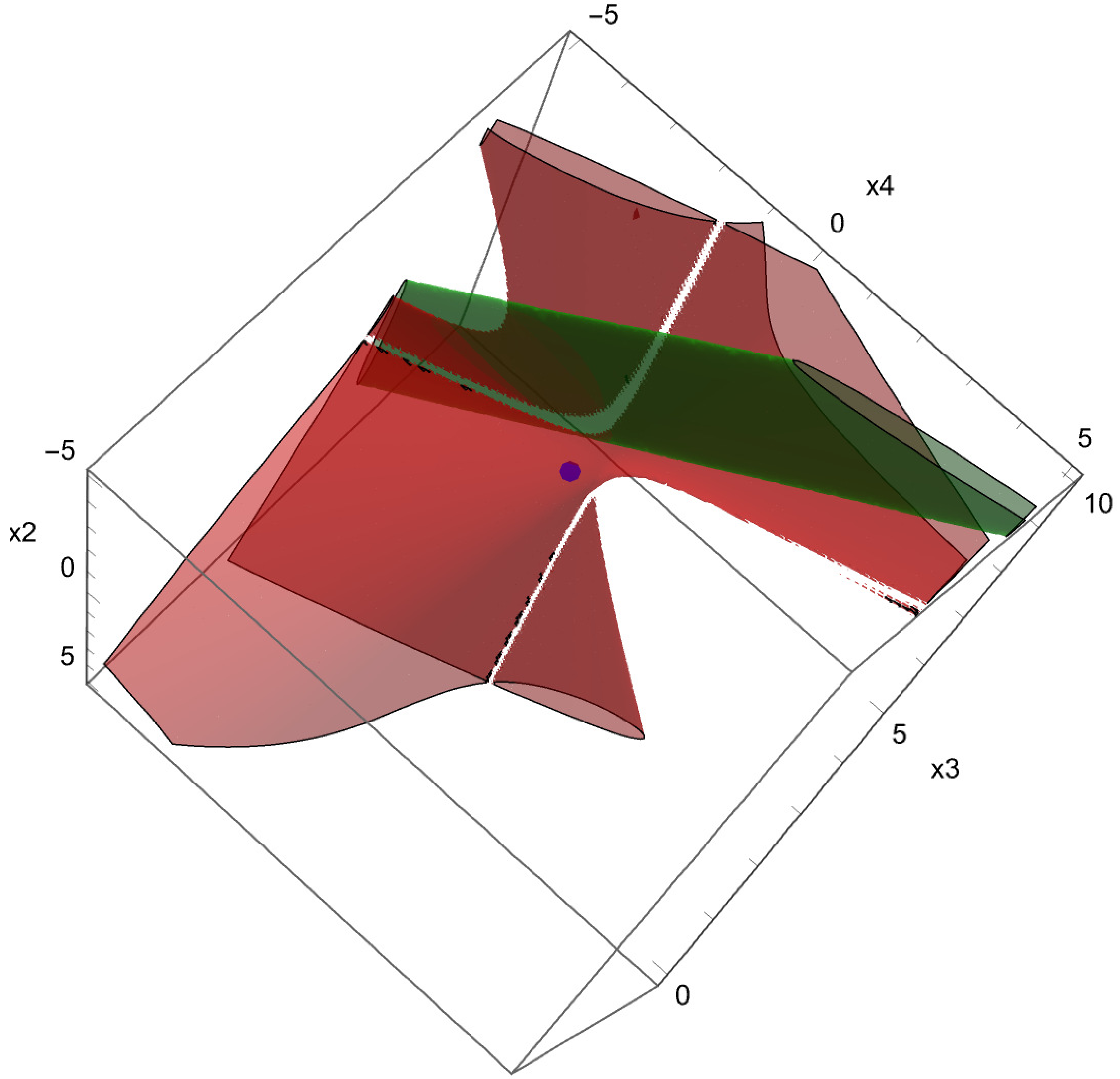

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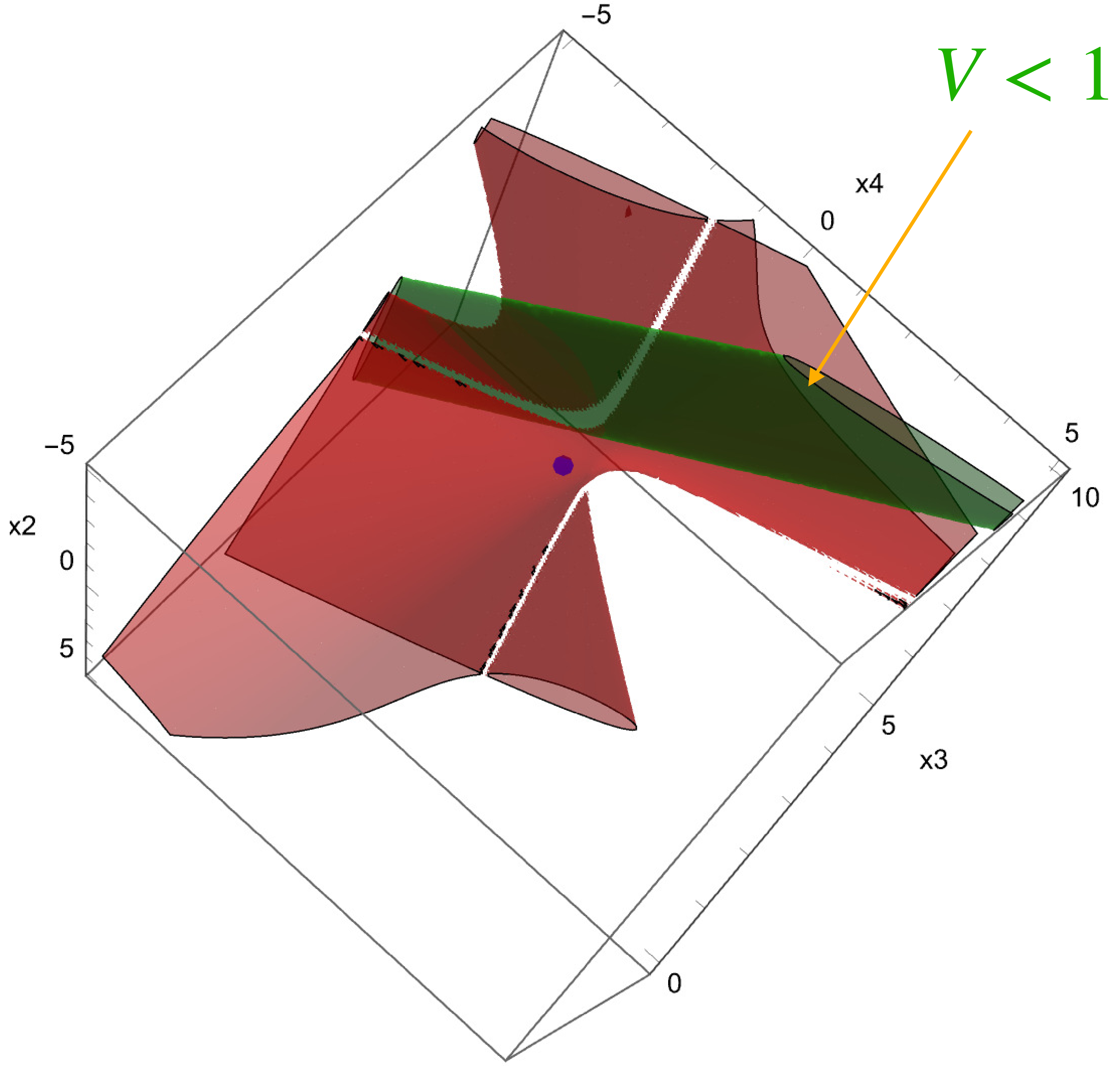

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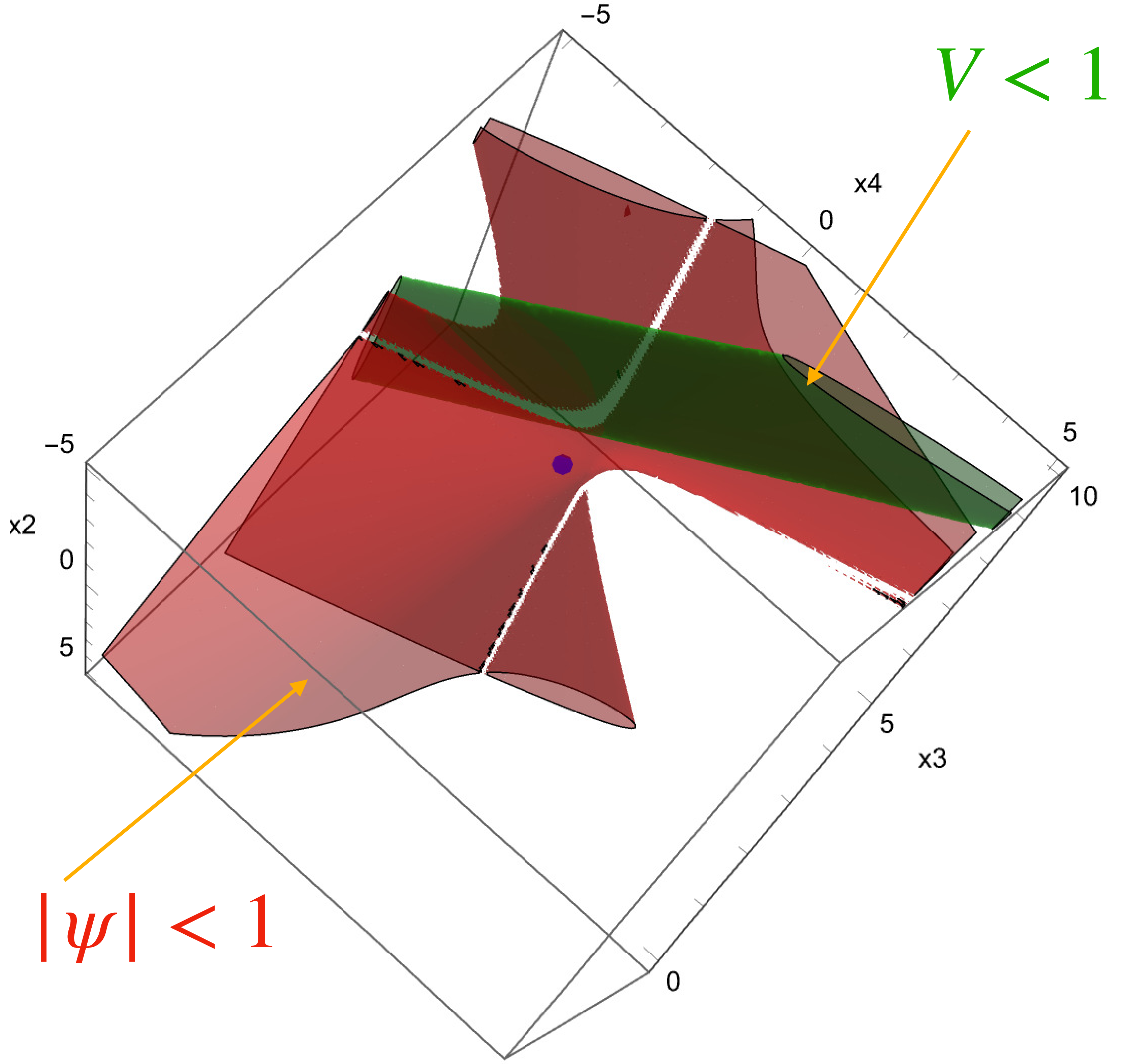

$V < 1$

F-term: $|\psi| < 1$

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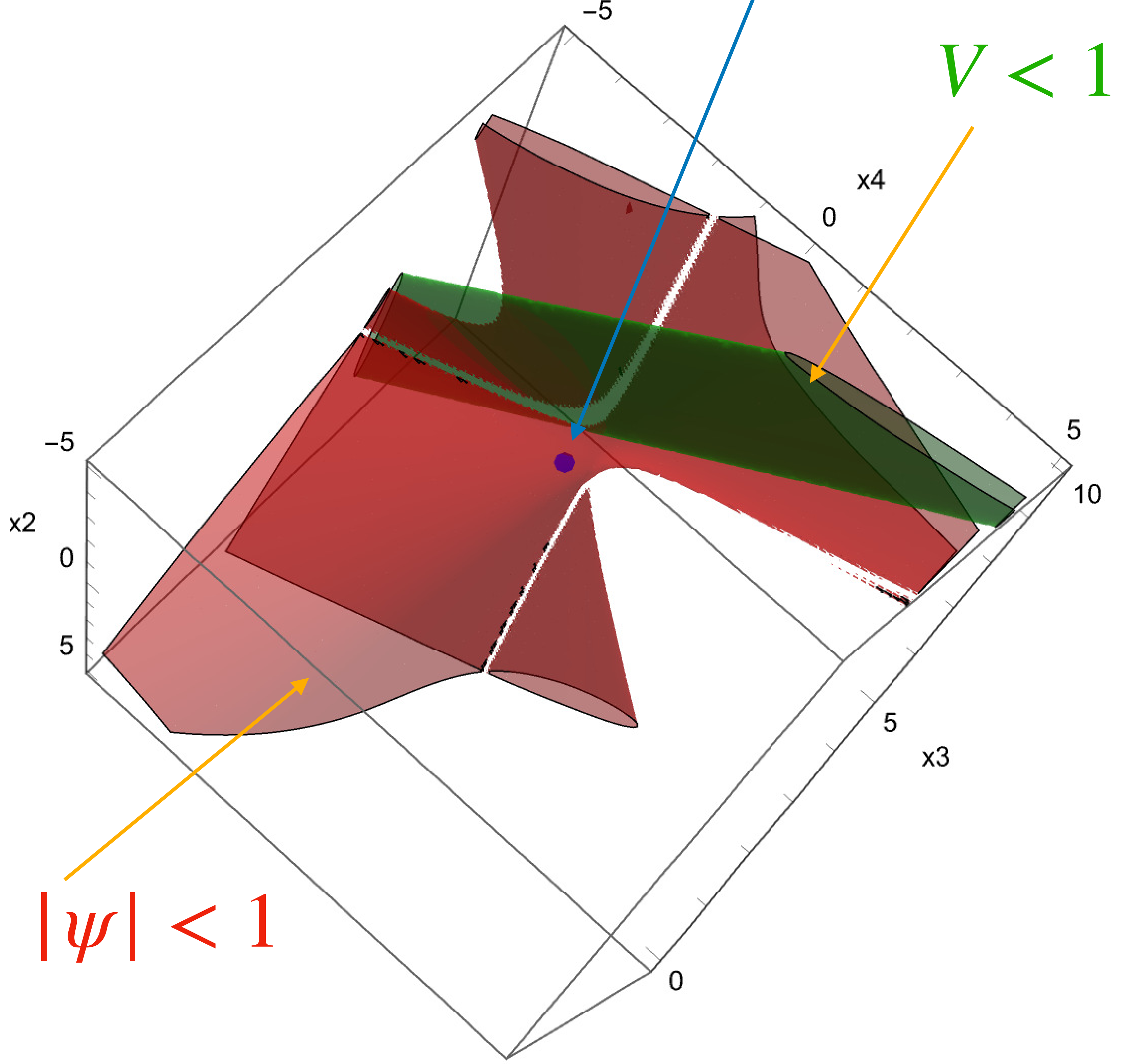

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Fermat-type One-parameter Models

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Manifolds

$$k = 5: \quad M = \left\{ x_i \in \mathbb{P}_{1,1,1,1,1} \,\middle|\, M_0 = x_0^5 + x_1^5 + x_2^5 + x_3^5 + x_4^5 = 0 \right\}$$

$$k = 6: \quad M = \left\{ x_i \in \mathbb{P}_{2,1,1,1,1} \,\middle|\, M_0 = 2x_0^3 + x_1^6 + x_2^6 + x_3^6 + x_4^6 = 0 \right\}$$

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Period Expressions

$$\varpi_0(\psi) = -\frac{\pi}{k} \sum_{n=1}^{\infty} \frac{1}{\Gamma(n) \prod_{i=0}^{4} \Gamma(1 - n\nu_i/k)} \frac{e^{\frac{i\pi(k-1)n}{k}}}{\sin\left(\frac{\pi n}{k}\right)} (\gamma\psi)^n \ ,$$

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Results

k	(n_0, n_1, m^0, m^1)	$\psi_{\min}$	V_F	$\lvert W_{\text{homo}}\rvert$	$\lvert\hat{\mathcal{W}}_0\rvert$	$\lvert\hat{\mathcal{W}}_1\rvert$
6	$(-1, 1, 7, 0)$	$0.392 + 0.679i$	**7.01**	**11.1**	0.655	2.98
6	$(-2, 2, -11, 3)$	$-0.392 - 0.679i$	**7.01**	11.1	**0.421**	1.55
6	$(0, -2, 5, 0)$	$-0.785 - 0.0000226i$	**7.01**	11.1	0.423	**1.49**
8	$(-2, -1, -1, 1)$	$0.250 - 0.250i$	25.5	**8.55**	191	1260
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8	$(-2, 1, -6, 2)$	$0.000463 - 0.802i$	**8.54**	11.1	99.5	**364**
10	$(2, -1, -1, -1)$	$-0.585 - 0.806i$	**2.79**	**9.61**	**1.42**	**2.19**

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Not amazing...

Two-parameter Model

Two-parameter Model

Manifold

$$p = x_1^8 + x_2^8 + x_3^4 + x_4^4 + x_5^4 - 8\psi x_1 x_2 x_3 x_4 x_5 - 2\phi x_1^4 x_2^4$$

Two-parameter Model

Period Expansion in $\left| \frac{8\zeta^4}{1 \pm \sqrt{\eta}} \right| < 1$

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$$\varpi_j(\zeta, \eta) = -\frac{1}{4} \sum_{m=1}^{\infty} \sum_{k=0}^{\infty} \alpha^{mj} \Delta(m) \Xi(k) \zeta^m \eta^k$$

Two-parameter Model

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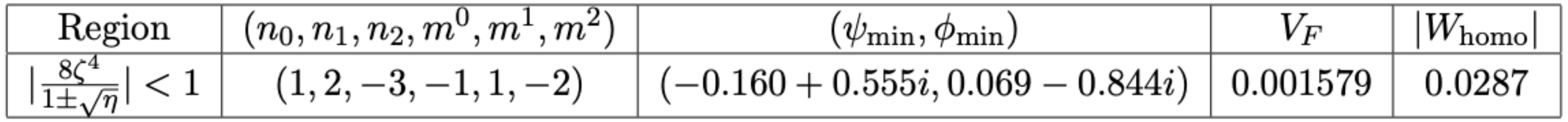

Region	$(n_0, n_1, n_2, m^0, m^1, m^2)$	$(\psi_{\min}, \phi_{\min})$	V_F	$\lvert W_{\text{homo}} \rvert$
$\left\lvert\frac{8\zeta^4}{1\pm\sqrt{\eta}}\right\rvert < 1$	$(1, 2, -3, -1, 1, -2)$	$(-0.160 + 0.555i, 0.069 - 0.844i)$	0.001579	0.0287

Period Expansion in $\left\lvert \dfrac{8\zeta^4}{1 \pm \sqrt{\eta}} \right\rvert < 1$

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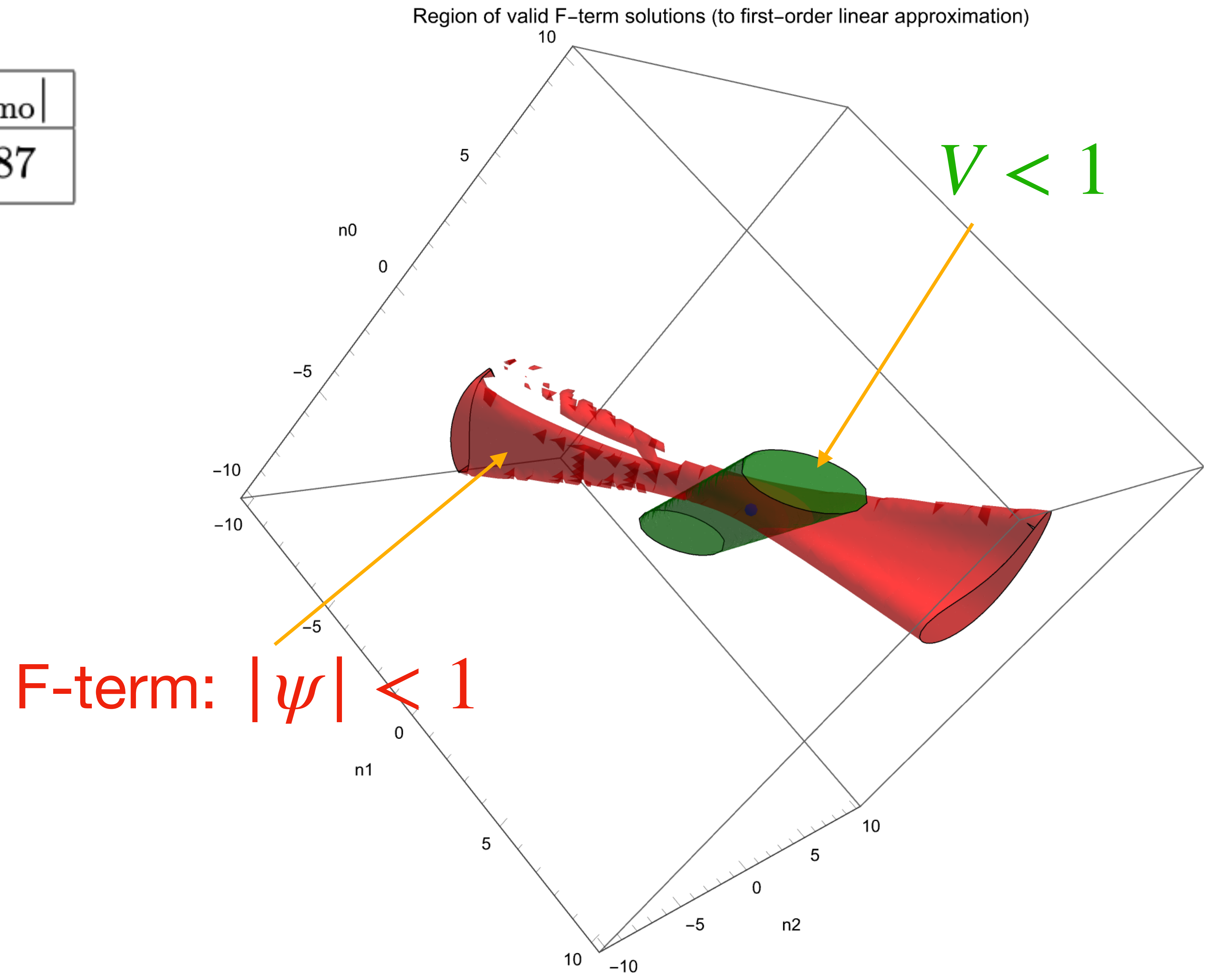


Region of valid F-term solutions (to first-order linear approximation)

$V < 1$

F-term: $|\psi| < 1$

Two-parameter Model

Period Expansion in $\left|\frac{8\zeta^4}{1\pm\sqrt{\eta}}\right| < 1$

Manifold

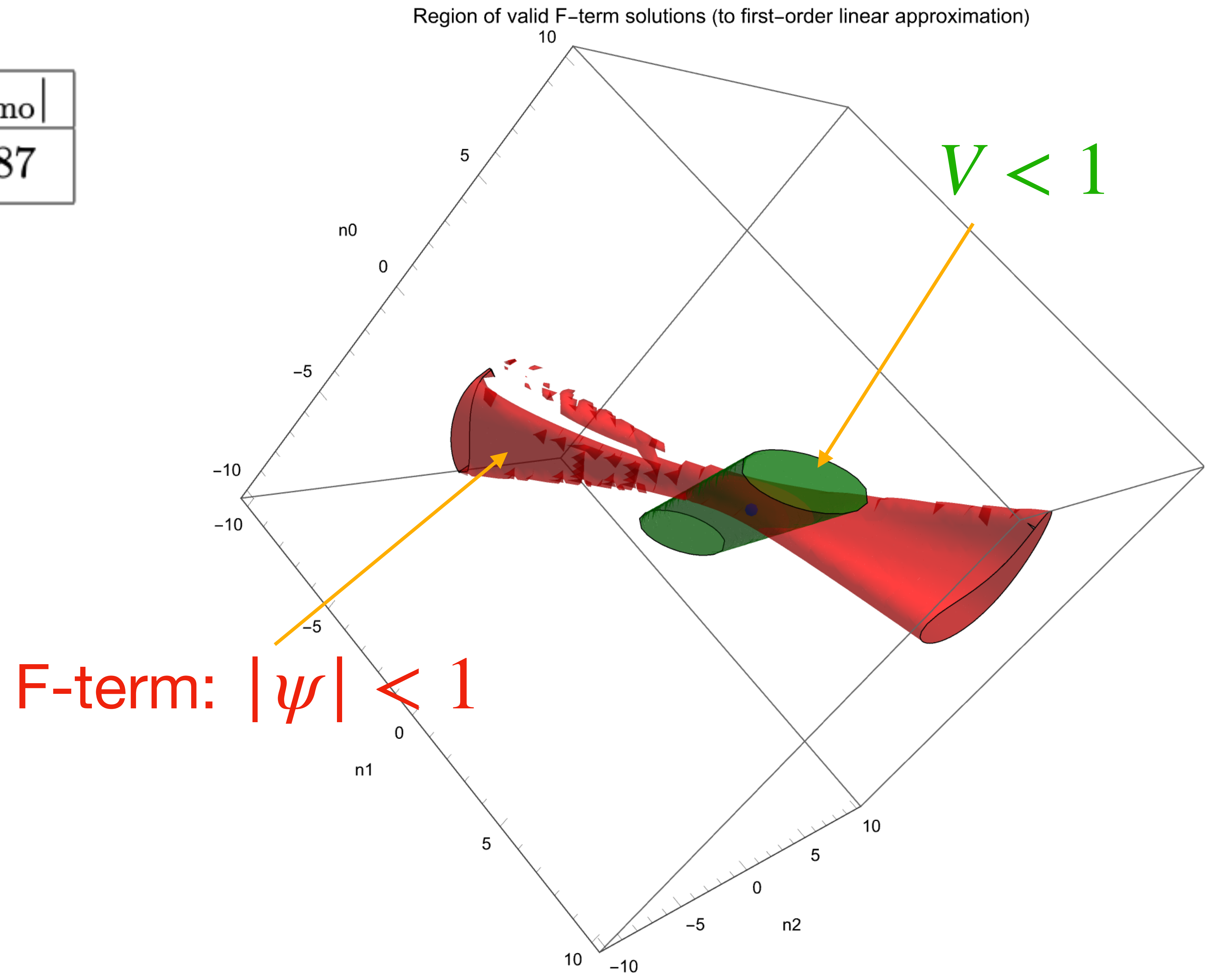
$$p = x_1^8 + x_2^8 + x_3^4 + x_4^4 + x_5^4 - 8\psi x_1x_2x_3x_4x_5 - 2\phi x_1^4x_2^4$$

$$\varpi_j(\zeta,\eta) = -\frac{1}{4}\sum_{m=1}^{\infty}\sum_{k=0}^{\infty}\alpha^{mj}\Delta(m)\Xi(k)\zeta^m\eta^k$$

Results

Region	$(n_0, n_1, n_2, m^0, m^1, m^2)$	$(\psi_{\min}, \phi_{\min})$	V_F	$\lvert W_{\text{homo}}\rvert$
$\lvert\frac{8\zeta^4}{1\pm\sqrt{\eta}}\rvert < 1$	$(1, 2, -3, -1, 1, -2)$	$(-0.160 + 0.555i, 0.069 - 0.844i)$	0.001579	0.0287

small W₀ seems possible!


Region of valid F−term solutions (to first−order linear approximation)

$V < 1$

F-term: $|\psi| < 1$

Conclusions

- Want to see if we can repeat KKLT-like scenario in heterotic string theory.
- No RR fluxes in heterotic means small-W_0 argument is not possible in heterotic.
- But W_0 seems possible in some models - need specific period structure!
- **Constraints on period structure can be in principle be derived!**

The Dream Scenario

The Dream Scenario

UV action

$$S_{\mathrm{UV}} = \int Dx \ldots$$

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UV action

$$S_{\text{UV}} = \int Dx \ldots$$

↓

4d effective action

$$S_{\text{4d}} = \int d^4x \, R + \ldots$$

The Dream Scenario

Integral

UV action

$$S_{\text{UV}} = \int Dx \ldots$$

4d effective action

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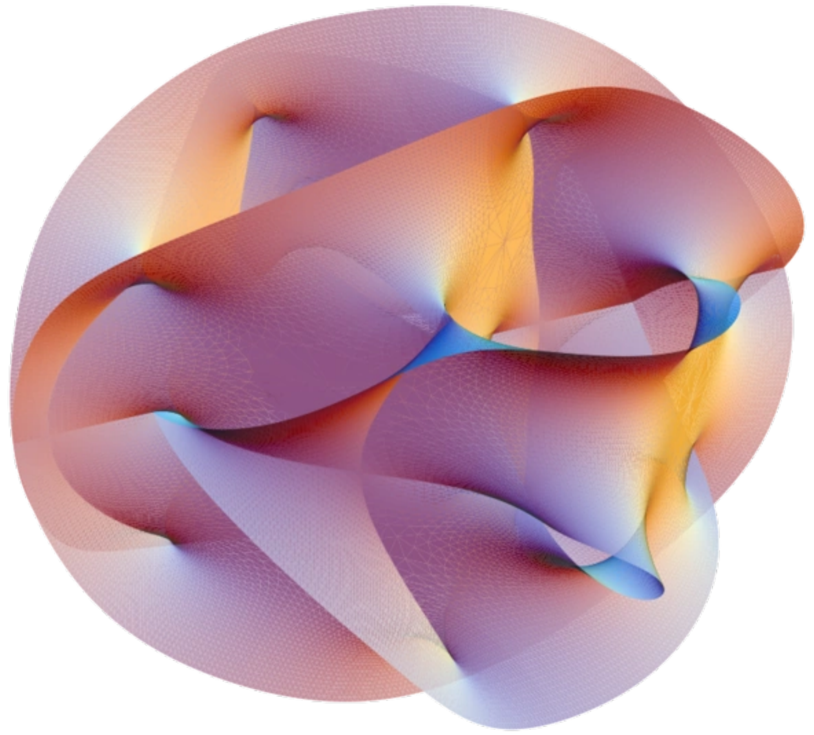
The Dream Scenario

Calabi-Yau Data
(geometry + topology)



UV action

Integral

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The Dream Scenario

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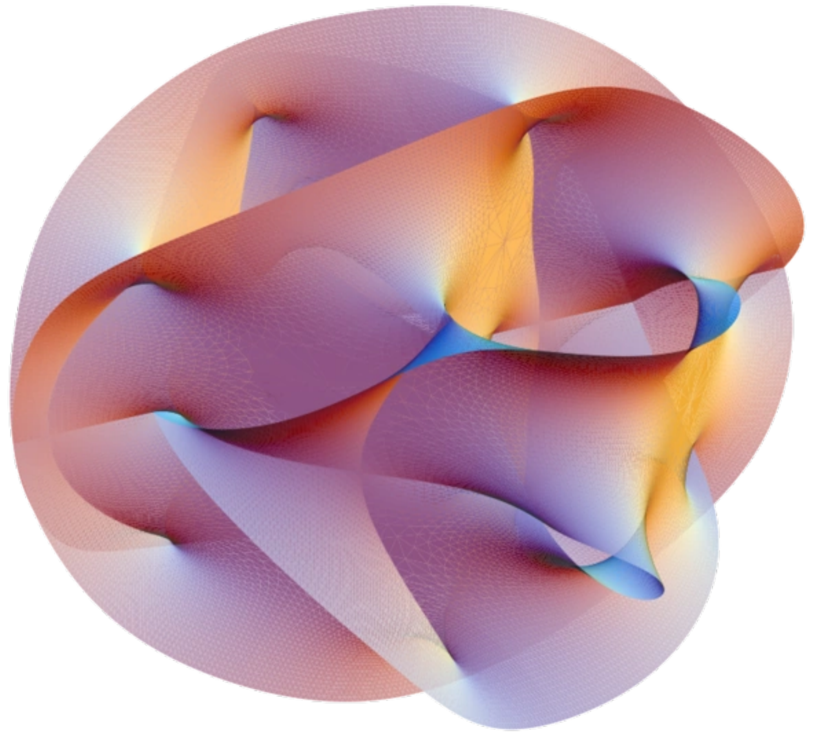

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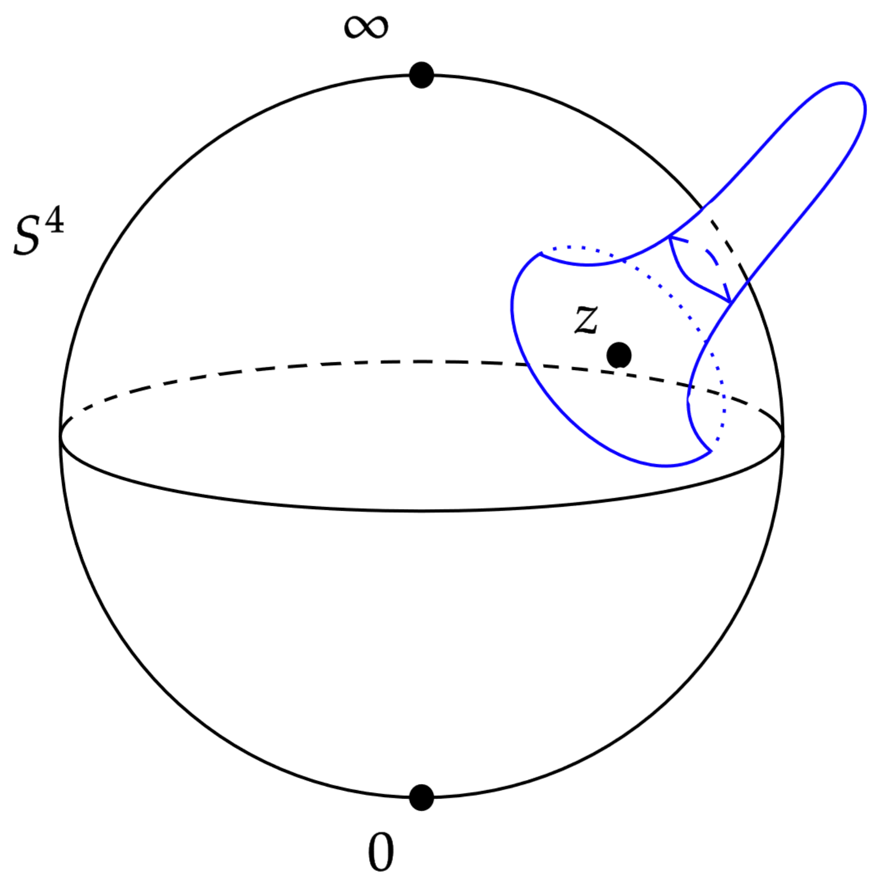
Integral

Non-perturbative effects



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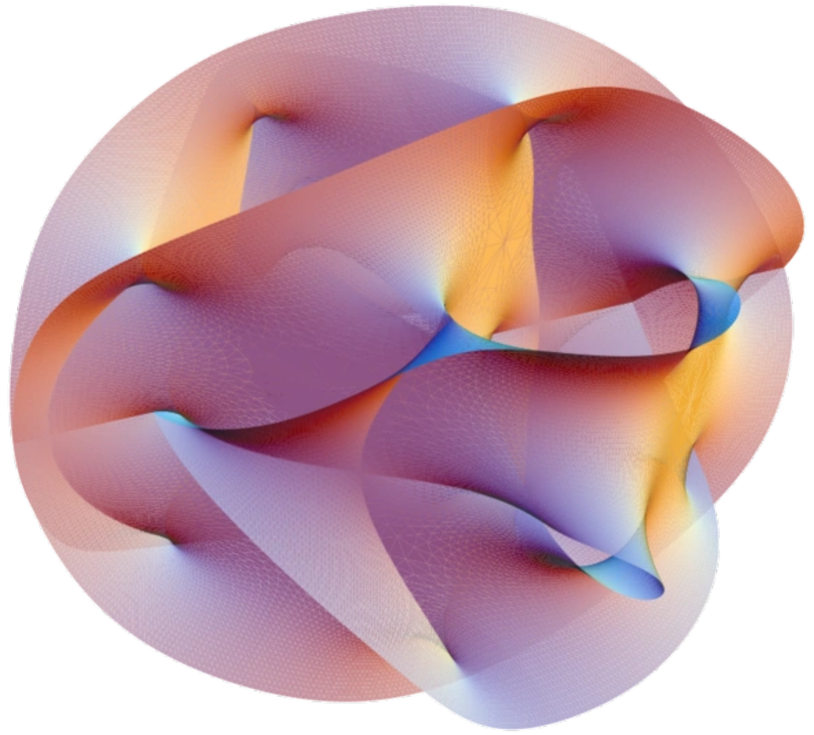

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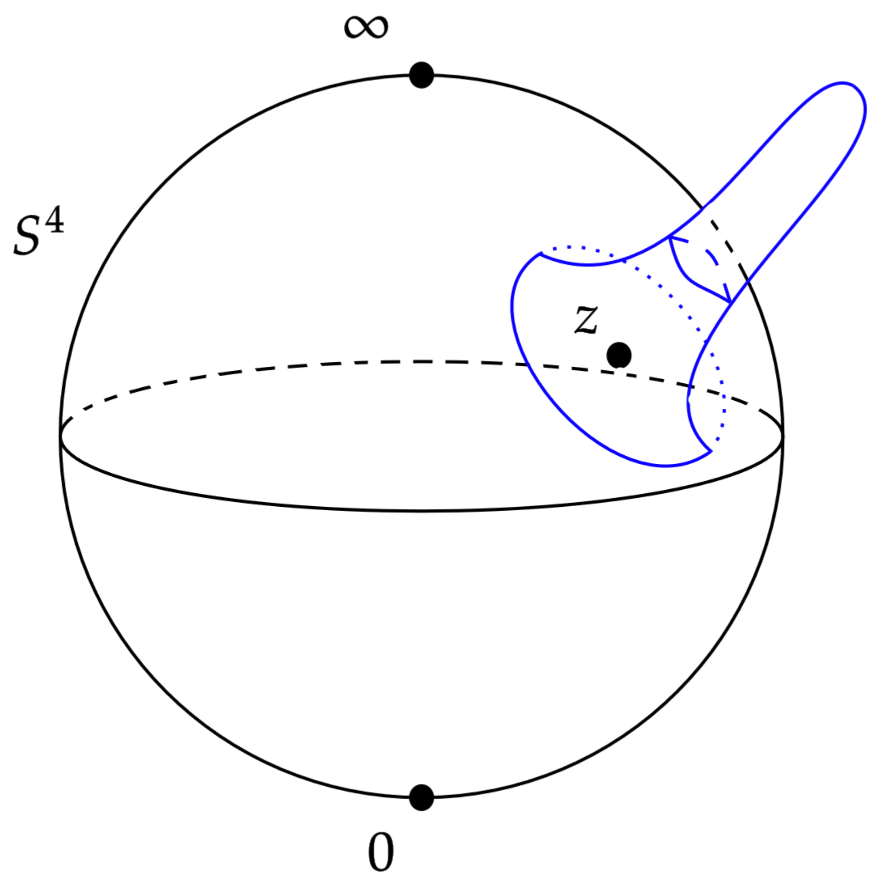
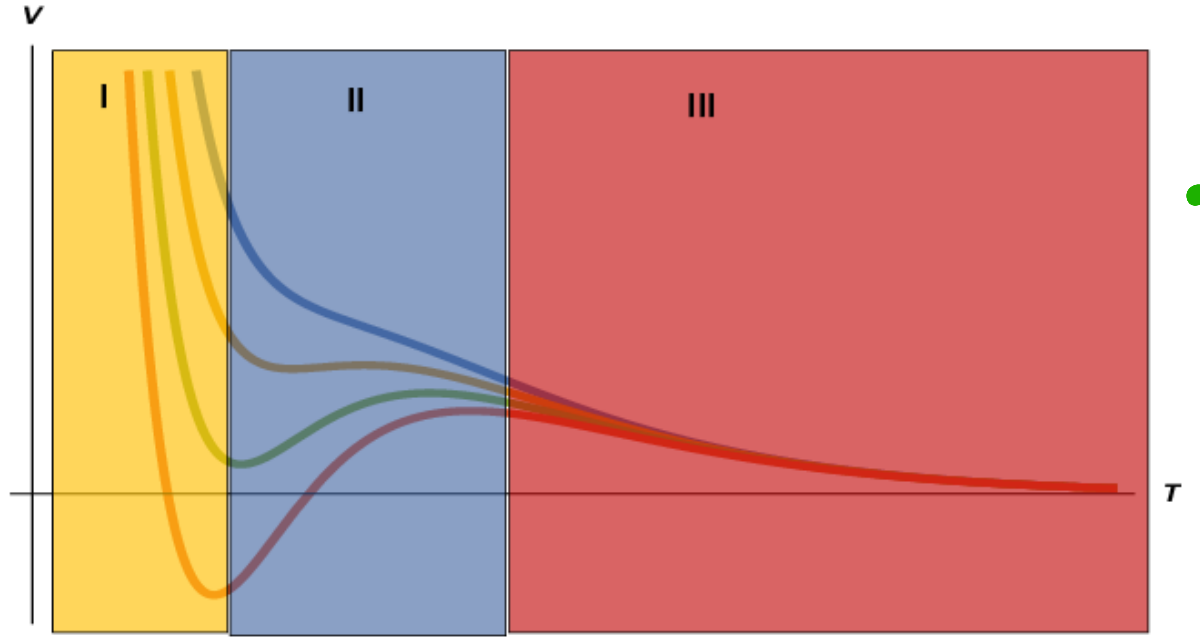


Moduli Stabilisation



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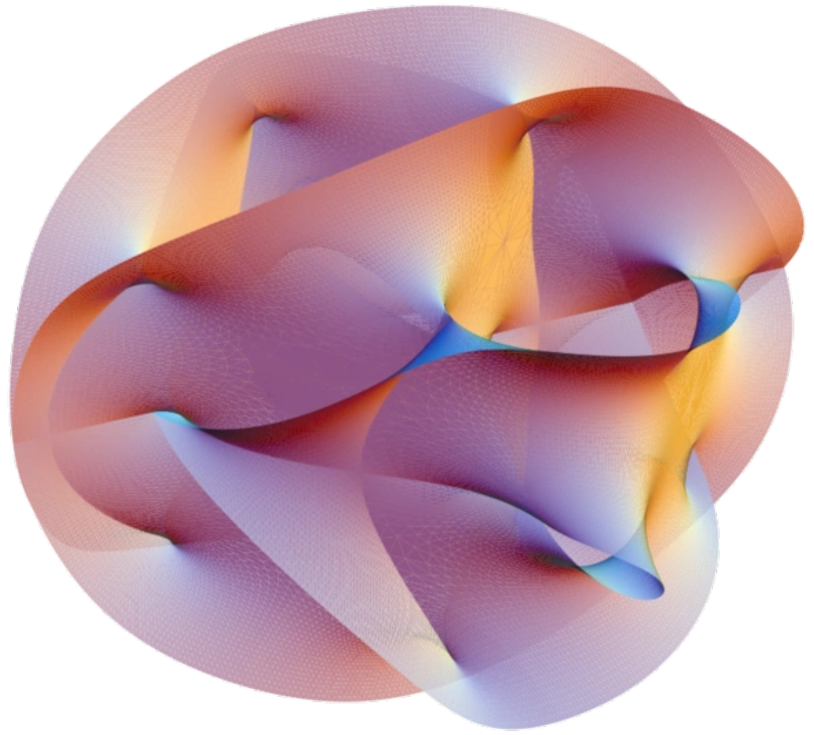
The Dream Scenario

Calabi-Yau Data (geometry + topology)



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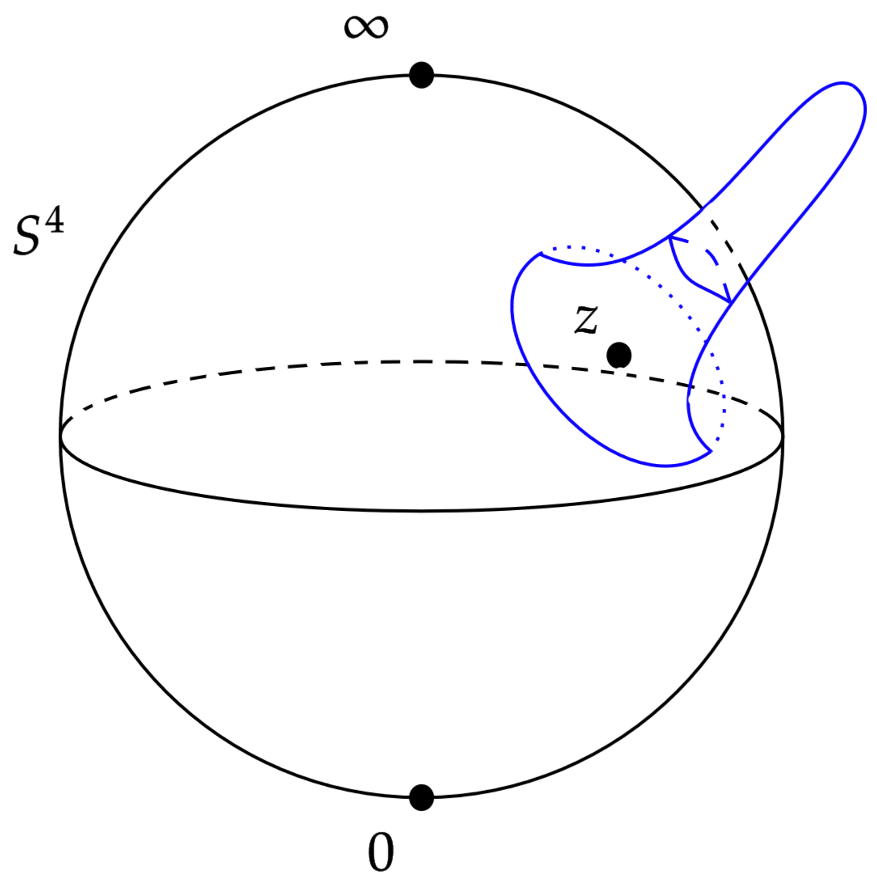

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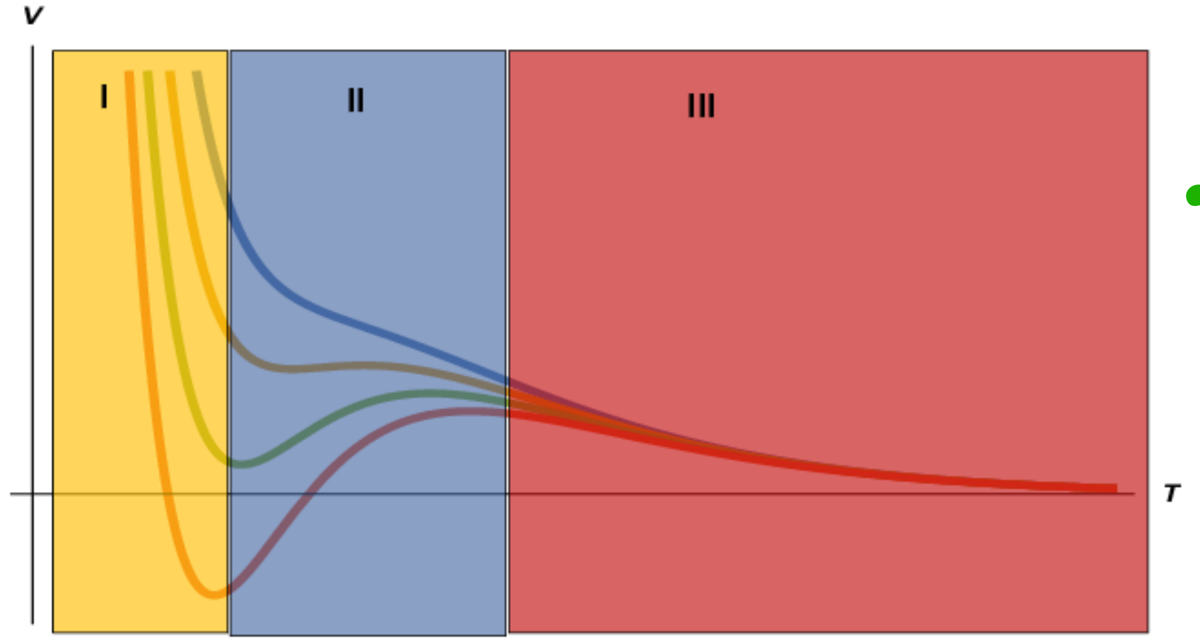

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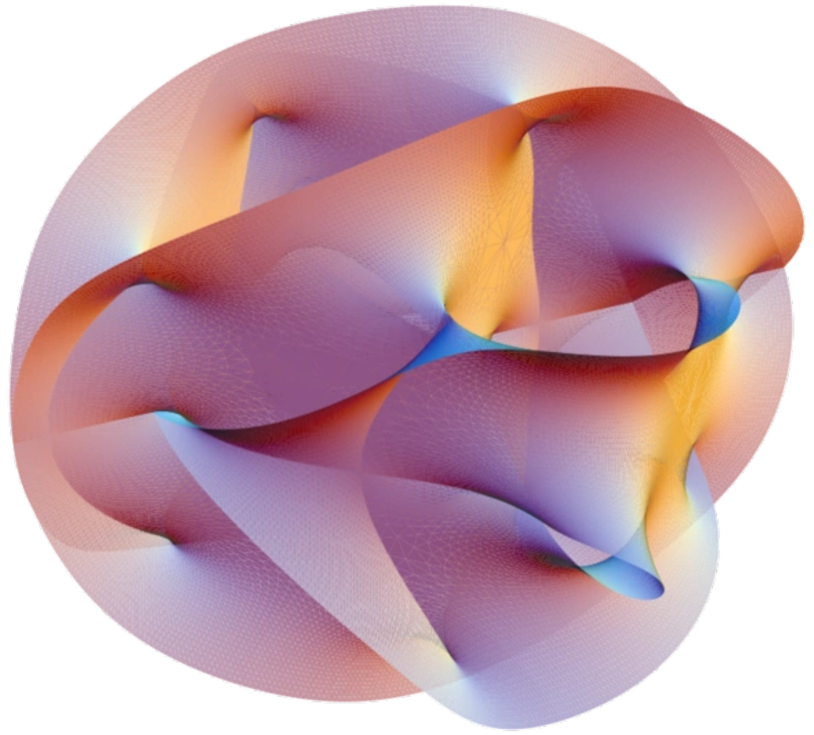

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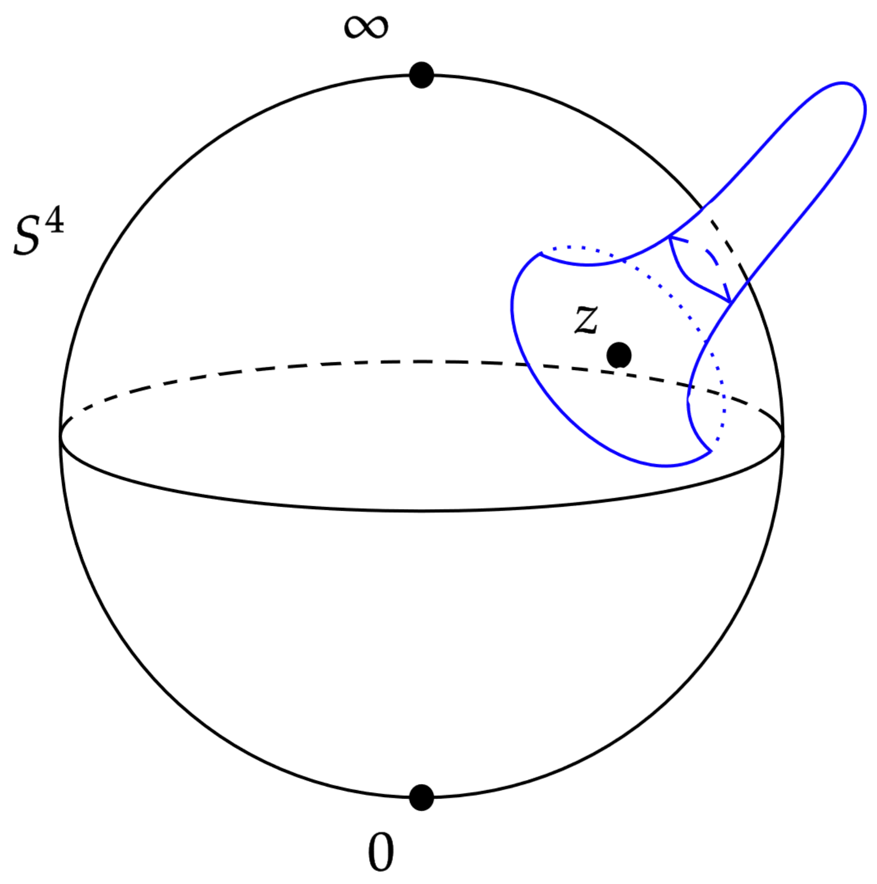

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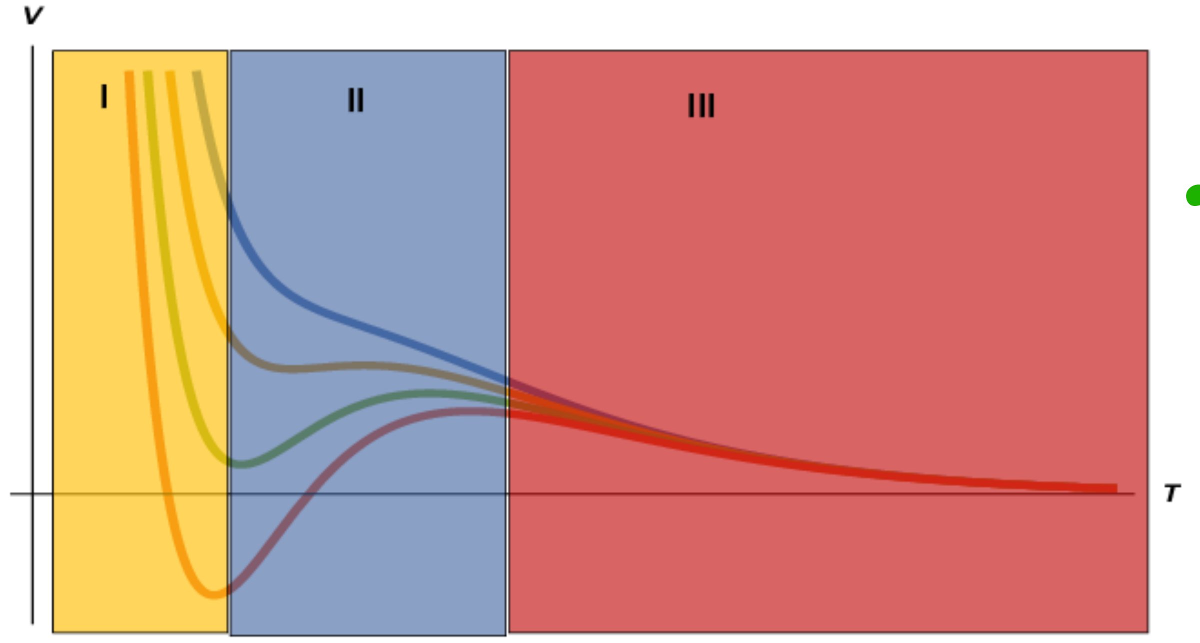

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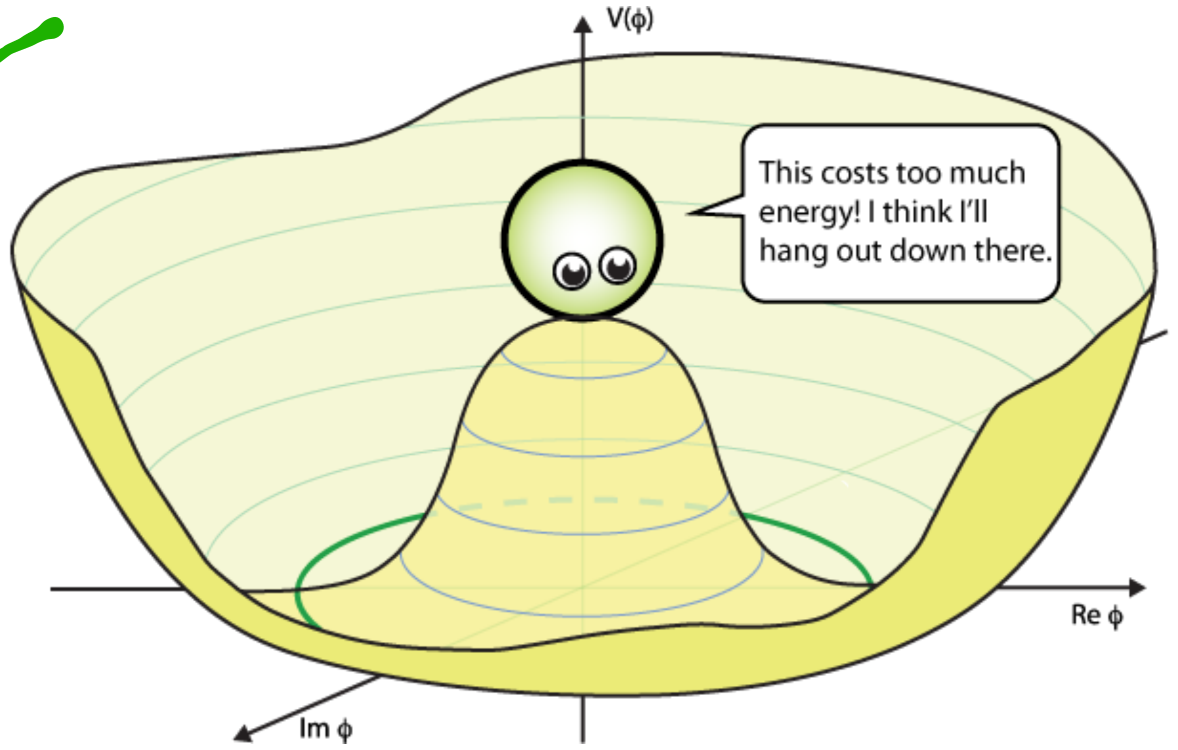
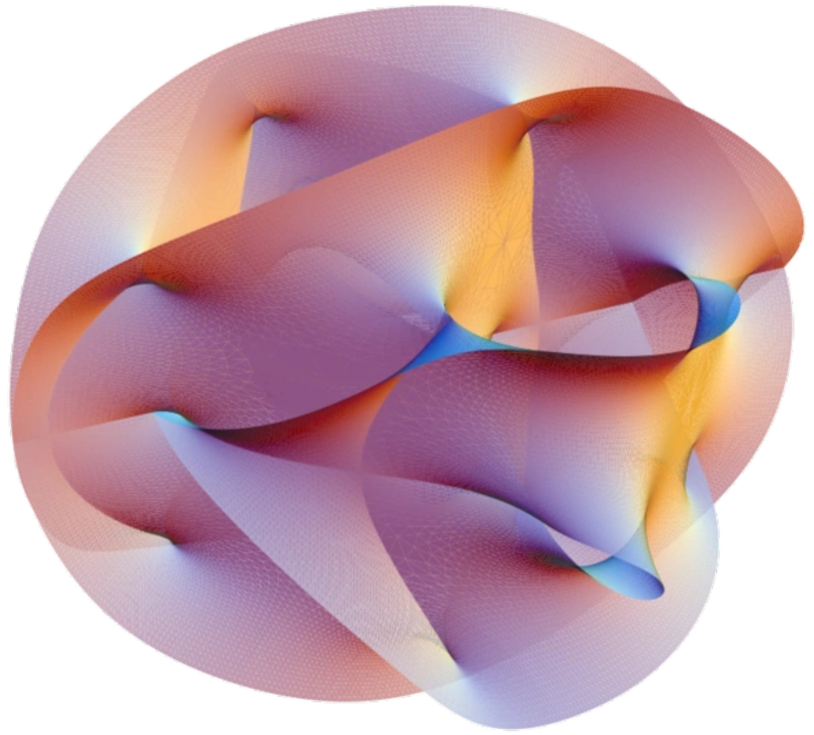
Electroweak Physics



The Dream Scenario

Calabi-Yau Data
(geometry + topology)



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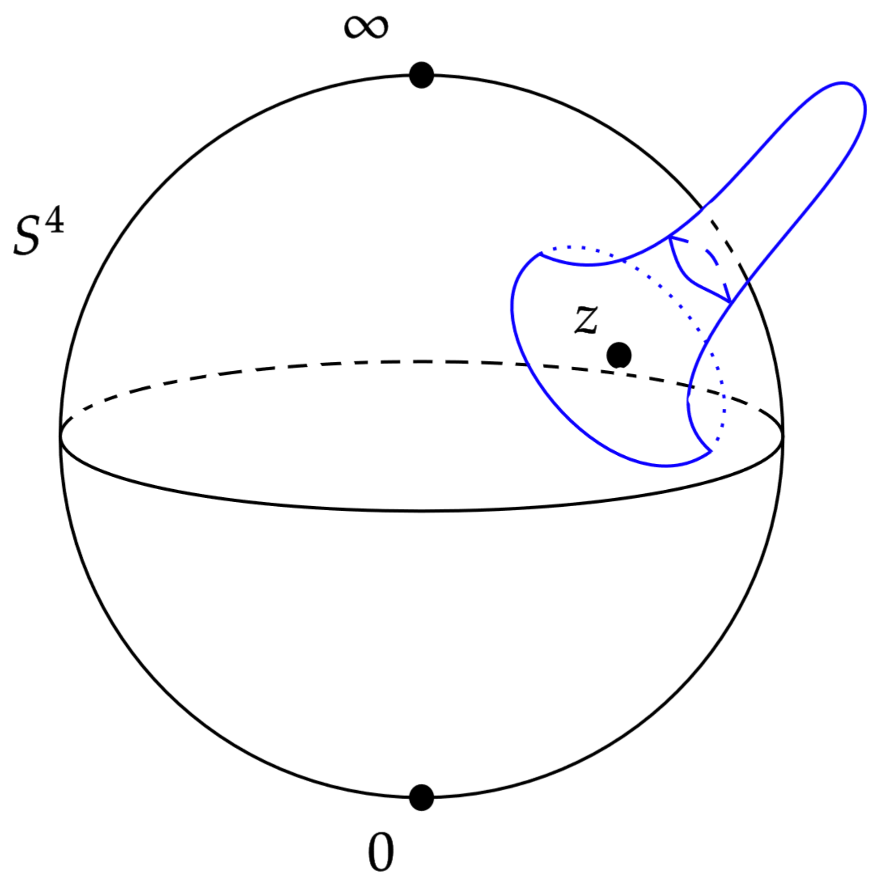

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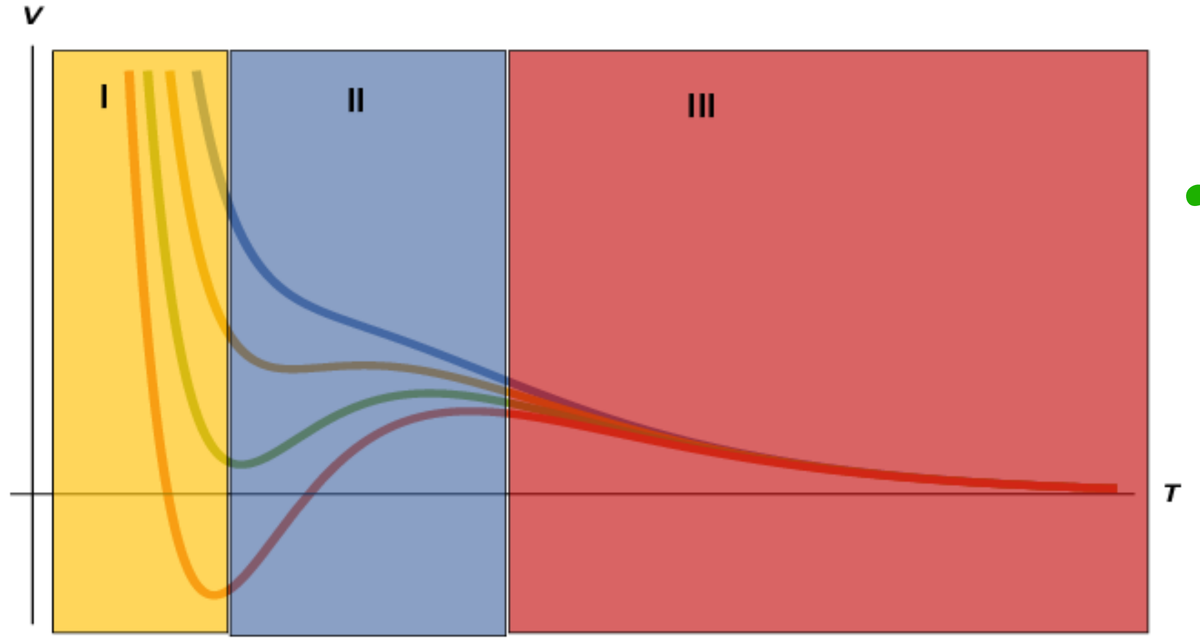

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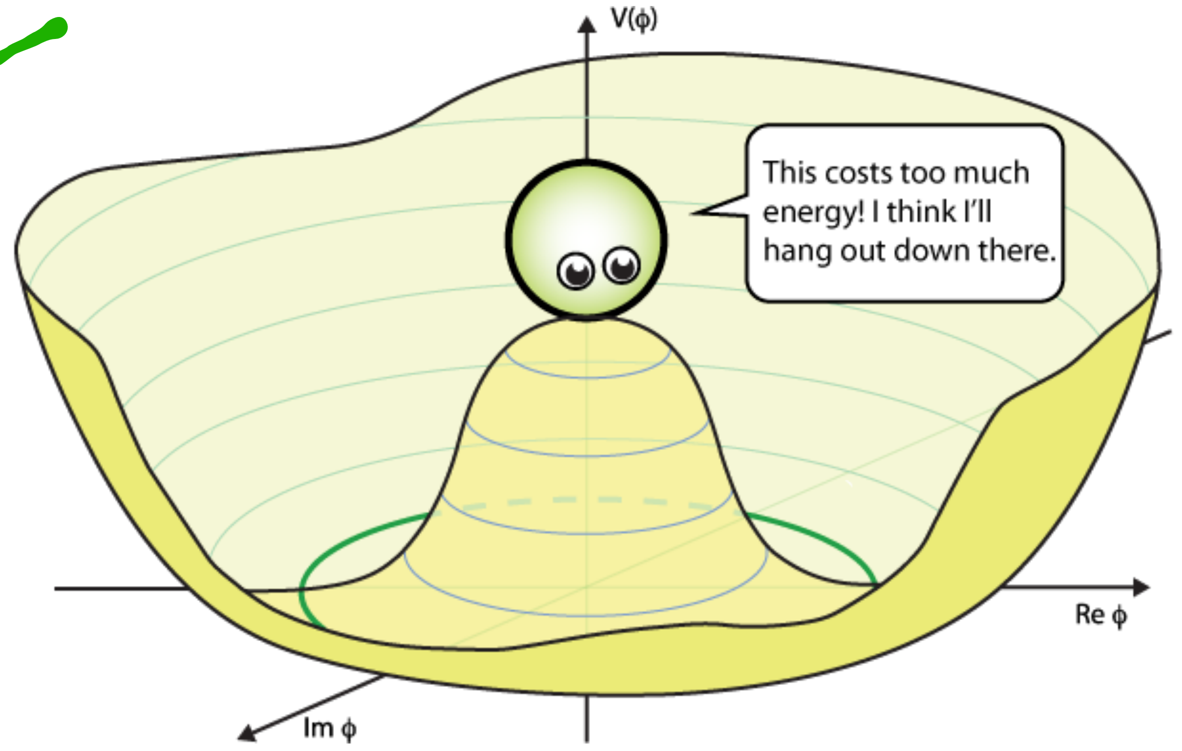

String Phenomenology

Outlook

- There is a lot more to do in string theory!
- **String model building is still hard - many computational and algebraic techniques needed.**
- Phenomenological issues like R-parity, electroweak symmetry breaking needs to be resolved.
- **Moduli stabilisation in heterotic string theory is difficult!**
- How often do ‘accidents’ occur?
- Are there general rules for small W_0?